On the $B \to K^* \ell\ell$ at low recoil

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based on arXiv:1606.00775 with S. Braß and G. Hiller

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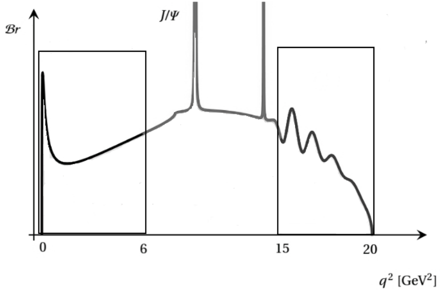
Motivation

- $b \to s\ell\ell$ - FCNC processes, loop induced and suppressed in the SM, potentially sensitive to possible BSM effects.
- Better precision enables us to probe higher energy scales, but also requires better understanding of the long distance dynamics within the SM (e.g. form factors, charm effects).
- $B \to K^*\mu\mu$ - rich angular information - large potential to diagnose the NP and long distance dynamics within SM

Two regions in q^2

- Low q^2 region - QCD factorization,
- high q^2 region - OPE (this talk).
- The limitations of both tools would be in the need for the revisit in the (near) future



$B \to K^*\ell\ell$ at high q^2

- Above J/ψ and $\psi(2S)$ resonances, the complicated intermediate charm states (presumably dominated by the wide charm resonances $J^{PC} = 1^{--}$) show up as wiggles in q^2 distributions. These effects have been anticipated from the 1970s.
- Theoretically controlled approach: Operator Product Expansion (OPE) in $1/Q^2$, $Q^2 \sim (q^2_{max}, m_b^2)$ of the non-local operator products

$$A_i^\mu \propto \frac{1}{q^2} \int d^4x e^{ikx} T\{\mathcal{O}_i(0) j^\mu(x)\} \quad (1)$$

- in terms of local operators [Grinstein, Pirjol (2004); Beylich, Buchalla, Feldmann (2011)]:

$$A^\mu = \sum_i C_i(q^2) Q_i^\mu . \quad (2)$$

- Leading power corrections from the orders α_s/m_b, m_c^4/Q^4 - only of order of few percents. Up to this precision charm contributions factorize, are universal and are absorbed into effective Wilson coefficients [Grinstein, Pirjol, (2004); Bobeth, Hiller, van Dyk, (2010)]

$$\begin{aligned}
\mathcal{C}_7^{\text{eff}} &= \mathcal{C}_7 - \frac{1}{3}\mathcal{C}_3 - \frac{4}{9}\mathcal{C}_4 - \frac{20}{3}\mathcal{C}_5 - \frac{80}{9}\mathcal{C}_6 + \frac{\alpha_s}{4\pi}\Big[(\mathcal{C}_1 - 6\mathcal{C}_2)A(q^2) - \mathcal{C}_8 F_8^{(7)}(q^2)\Big], \\
\mathcal{C}_9^{\text{eff}} &= \mathcal{C}_9 + \frac{1}{2}h(q^2,0)\Big[\frac{8}{3}\mathcal{C}_1 + 2\mathcal{C}_2 + 11\mathcal{C}_3 - \frac{4}{3}\mathcal{C}_4 + 104\mathcal{C}_5 - \frac{64}{3}\mathcal{C}_6\Big] \\
&\quad + \frac{8}{3}\frac{m_c^2}{q^2}\Big[\frac{4}{3}\mathcal{C}_1 + \mathcal{C}_2 + 6\mathcal{C}_3 + 60\mathcal{C}_5\Big] \\
&\quad + \frac{\alpha_s}{4\pi}\Big[\mathcal{C}_1\big(B(q^2) + 4C(q^2)\big) - 3\mathcal{C}_2\big(2B(q^2) - C(q^2)\big) - \mathcal{C}_8 F_8^{(9)}(q^2)\Big] \\
&\quad - \frac{1}{2}h(q^2, m_b^2)\Big[7\mathcal{C}_3 + \frac{4}{3}\mathcal{C}_4 + 76\mathcal{C}_5 + \frac{64}{3}\mathcal{C}_6\Big] + \frac{4}{3}\Big[\mathcal{C}_3 + \frac{16}{3}\mathcal{C}_5 + \frac{16}{9}\mathcal{C}_6\Big].
\end{aligned} \tag{3}$$

- OPE expected to provide good description of charm effects within binned observables.
- Quark-hadron duality violations enter at some level, but their magnitude is not known within the OPE itself.

Plan

- Compare the predictions of the OPE with experimental data for angular observables and branching fractions at low recoil.
- Use the Krüger-Sehgal (KS) data driven model of local q^2 distributions, to estimate the uncertainties (limitations of the OPE) for a chosen binning, independently of the underlying electro-weak model (SM or BSM)

Using OPE - Angular Observables

- The "improved" Isgur-Wise relations and OPE lead to universal short-distance couplings [Bobeth, Hiller, van Dyk (2010); Hambrock, Hiller, (2012); Hambrock, Hiller, Schacht, Zwicky (2013)]

$$\begin{aligned} \mathcal{A}_\perp^{L,R} &= +i\, C^{L,R} f_\perp,\ \mathcal{A}_{\|,0}^{L,R} = -i\, C^{L,R} f_{\|,0} \ \text{ where} \\ C^{L,R} &= \mathcal{C}_9^{\text{eff}} \mp \mathcal{C}_{10} + \kappa \frac{2\hat{m}_b}{\hat{s}} \mathcal{C}_7^{\text{eff}} \end{aligned} \tag{4}$$

- Then the observables F_L, S_3, S_4 turn out independent of $C^{L,R}$ and only depend on form factors (within OPE, IW and no RH quark currents assumption). [Hambrock, Hiller, (2012); Hambrock, Hiller, Schacht, Zwicky, (2013)]. If universal, also the wiggles cancel in $F_L, S_{3,4}$ for infinitesimal bin size, their appearance signals the non-universal effects.

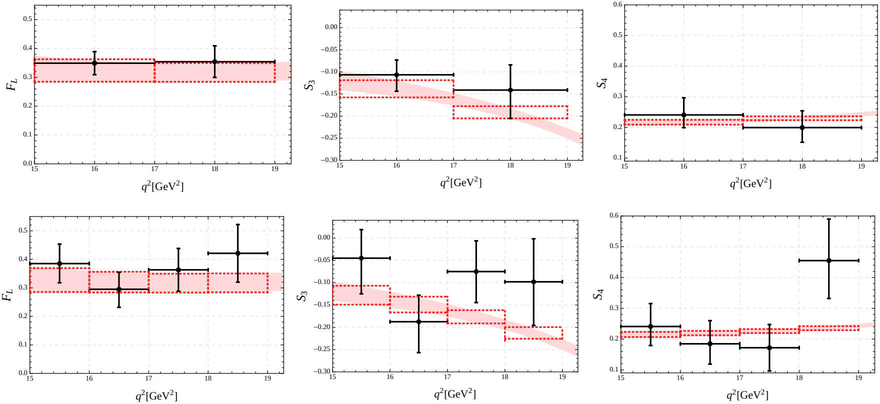
One can then test the SM with $d\mathcal{B}/dq^2$ and the observables S_5, A_{FB}:

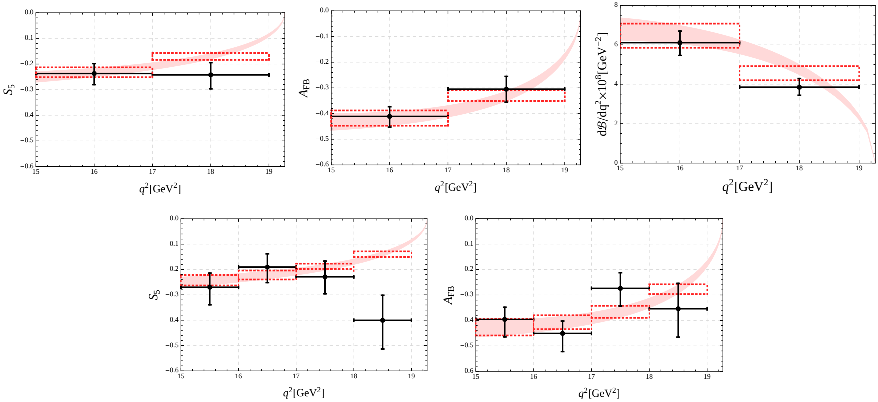
$$S_5 = \frac{3\sqrt{2}}{2}\frac{\rho_2(q^2) f_0 f_\perp}{\rho_1(q^2)\left(f_0^2 + f_\perp^2 + f_\parallel^2\right)}, \quad A_{FB} = \frac{3\rho_2(q^2) f_\parallel f_\perp}{\rho_1(q^2)\left(f_0^2 + f_\perp^2 + f_\parallel^2\right)}, \tag{5}$$

where

$$\begin{aligned} \rho_1(q^2) &\equiv \frac{1}{2}(|C^R|^2 + |C^L|^2) = \left|\mathcal{C}_9^{\text{eff}} + \kappa\frac{2m_b m_B}{q^2}\mathcal{C}_7^{\text{eff}}\right|^2 + |\mathcal{C}_{10}|^2, \\ \rho_2(q^2) &\equiv \frac{1}{4}(|C^R|^2 - |C^L|^2) = Re\left[\left(\mathcal{C}_9^{\text{eff}} + \kappa\frac{2m_b m_B}{q^2}\mathcal{C}_7^{\text{eff}}\right)\mathcal{C}_{10}^*\right]. \end{aligned} \tag{6}$$

F_L, S_3, S_4: OPE + ($\mathcal{C}'_{9,10} = 0$) - comparison with Experiment LHCb, 1512.04442



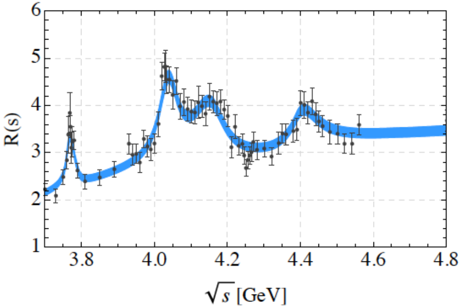


- Only S_4, S_5 show some tension with the OPE in the highest 1GeV^2 bin.

Krueger-Sehgal parametrisation

- Krüger-Sehgal (KS) model [Kruger, Sehgal, (1996)] - data from $e^+e^- \to$ hadrons is used to obtain charm vacuum polarization, which is then plugged in the $B \to K^*$ and corrected with fudge factors η_c.
- Extraction of the charm vacuum polarization $h_c(q^2)$ from the $e^+e^- \to h_i$ data [BES Collaboration, (2007)]

$$R(s) = \frac{\sigma^{(e^+e^- \to h_i)}(s)}{\sigma^{(e^+e^- \to \mu^+\mu^-)}(s)}. \tag{7}$$

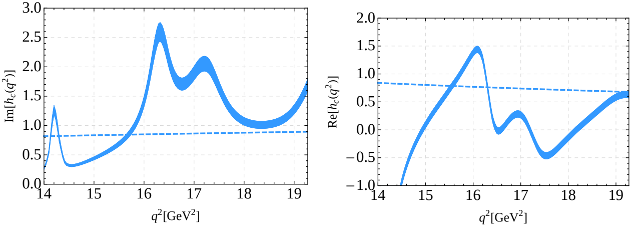


Krueger-Sehgal parametrisation

- Using the optical theorem and the dispersion relation:

$$
\begin{aligned}
&\mathrm{Im}[\mathcal{A}(e^+e^- \to h \to e^+e^-)] = s\sigma(e^+e^- \to h)(s), \\
&\mathrm{Im}[h_c(s)] = \frac{\pi}{3} R_c(s), \\
&\mathrm{Re}[h_c(s)] = \mathrm{Re}[h_c(s_0)] + \frac{s - s_0}{\pi} P \int_{t_0}^{\infty} \frac{ds'}{(s' - s)(s' - s_0)} \mathrm{Im}[h_c(s')].
\end{aligned} \tag{8}
$$

Krüeger-Sehgal parametrisation ctd.



$$\mathcal{C}_9^{\rm eff}(q^2) = \mathcal{C}_9 + 3a_2\,\eta_c\,h_c(q^2) + \ldots \tag{9}$$

[Chetyrkin, Misiak, Münz (1996)], Bobeth, Misiak, Urban, (2000),

$$a_2 = \frac{1}{3}\left(\frac{4}{3}\mathcal{C}_1 + \mathcal{C}_2 + 6\mathcal{C}_3 + 60\mathcal{C}_5\right) = 0.2 \text{ at NNLO at the } b\text{-mass scale.} \tag{10}$$

- Introduce "fudge function" $\eta_c \equiv \eta_c(K_j^*, q^2)$, $j = \perp, \|, 0$, that corrects for effects beyond NFA ($|\eta_c = 1|$ and universal). We take $\eta_c(j) \in \mathbb{R}$, more generally $\eta_c(j) \in \mathbb{C}$.

Krüeger-Sehgal parametrisation ctd.

- $B \to K\mu^+\mu^-$ wiggles observed in the high q^2 measured by LHCb [LHCb, 1307.7595]. [Lyon, Zwicky, (2014)] : require large fudge factor ~ -2.5; one can also probe ψ_i dependent correction factors.
- We expect the correction factor(s) for $B \to K^*\psi_i$ to differ wrt the $B \to K\psi_i$, e.g.

$$|\eta_{J/\psi(\psi(2S))\,K^*}| \simeq 1, \;\; \text{while } |\eta_{J/\psi(\psi(2S))\,K}| \simeq 1.5. \tag{11}$$

- We take in our analysis $\eta's$ constant but polarization dependent. With more experimental information, we could be more general.
- Kinematic relations at the endpoint ($q^2 = q^2_{\rm max}$) [Hiller, Zwicky, (2013)],

$$\mathcal{A}_\parallel = -\sqrt{2}\mathcal{A}_0, \quad \mathcal{A}_\perp = 0, \tag{12}$$

and our *ansatz* $\mathcal{C}_9^{{\rm eff},i}(q^2) = \mathcal{C}_9 + 3a_2\,\eta_i\,h_c(q^2) + \ldots\,(i = 0, \|, \perp)$, imply:

$$\eta_0 = \eta_\parallel,\, \eta_\perp \to \text{not constrained} \tag{13}$$

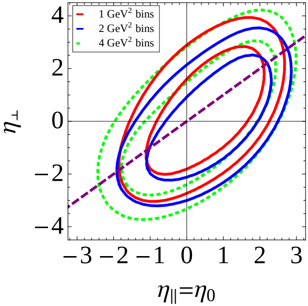
since $f_\perp(q^2_{\rm max}) = 0$.

Krüeger-Sehgal parametrisation ctd.

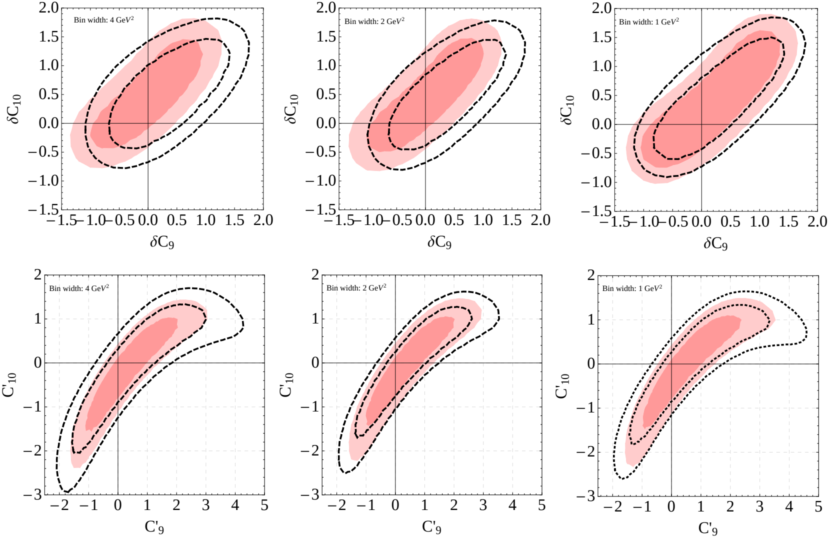
- The observables $S_{7,8,9}$ provide null tests for the hadronic universality (vanish in the OPE), (see also Bobeth, Hiller, van Dyk (2012)), for example:

$$S_8 \sim f_0 f_\perp (\tilde{\mathcal{C}}_9^{\text{eff}} + \kappa \frac{2 m_b m_B}{q^2} \mathcal{C}_7^{\text{eff}}) a_2 \text{Im}[h_c(q^2)(\eta_0 - \eta_\perp)] \,. \tag{14}$$

- We perform the simultaneous fit for $\eta_{0,\perp}, \mathcal{C}_9, \mathcal{C}_{10}$



OPE vs Resonance model for 3 different binnings[1]



[1] $\chi^2/\mathrm{dof} = (1.3, 0.8, 1.3)$ for OPE and $(1.0, 0.6, 1.2)$ for KS

- Differences between shaded areas (OPE) and dashed lines (KS) can serve as a binning dependent systematic uncertainty for OPE.
- To estimate the uncertainties of the OPE predictions for a given binning, we suggest the ratios:

$$\epsilon_1 = \frac{\int_{bin} \rho_1^{KS} dq^2}{\int_{bin} \rho_1^{OPE} dq^2}, \quad \epsilon_2 = \frac{\int_{bin} \rho_2^{KS} dq^2}{\int_{bin} \rho_2^{OPE} dq^2}, \quad \epsilon_{12} = \frac{\int_{bin} \rho_2^{KS} dq^2}{\int_{bin} \rho_2^{OPE} dq^2} \cdot \frac{\int_{bin} \rho_1^{OPE} dq^2}{\int_{bin} \rho_1^{KS} dq^2}. \quad (15)$$

bin [GeV^2]	$15-19$	$15-17$	$17-19$	$15-16$	$16-17$	$17-18$	$18-19$
ϵ_1	(0.85,1.16)	(0.81,1.30	(0.87,1.03)	(0.76,1.20)	(0.84,1.38)	(0.84,1.03)	(0.86,1.05)
ϵ_2	(0.82,1.0)	(0.74,1.13)	(0.85,0.91)	(0.71,1.17)	(0.78,1.08)	(0.76,0.95)	(0.84,0.97)
ϵ_{12}	(0.86,1.05)	(0.87,1.05)	(0.84,1.05)	(0.95,1.06)	(0.78,1.05)	(0.75,1.05)	(0.93,1.05)

Table: Ratios ϵ_k for different q^2-bins and 1σ ranges of parameters $\eta_0, \eta_\perp$ and $\mathcal{C}_{9,10}$. The coefficients $\mathcal{C}'_{9,10} \to 0$.

- This suggests that OPE performs better at endpoint bins than at the lower q^2 bins and the bin of maximal size $(15-19)\text{GeV}^2$.

Conclusions

- So far, good performance of the SM+OPE, although large BSM effects are allowed.
- Small binnings and more precise data is going to probe the limits of the OPE.
- $B \to K^* \ell\ell$ provides large(r) number of angular observables to hopefully disentangle the SD and LD effects.
- CP-averages $S_{7,8,9}$ important for testing the hadronic universality of charm effects.
- Consistency between fits to Wilson coefficients between high q^2, low q^2 and inclusive decays are important to decide the fate of the $B \to K^* \mu\mu$ anomaly.

Backup slides - The notation

Let us review the notation and the conventions.

- The effective Hamiltonian:

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i \mathcal{C}_i(\mu)\mathcal{O}_i + h.c \tag{16}$$

- We use CMM basis $\mathcal{O}_1 - \mathcal{O}_8$ [Chetyrkin, Misiak, Münz (1996)], e.g.

$$\mathcal{O}_1^c = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma_\mu T^a b_L), \quad \mathcal{O}_2^c = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L). \tag{17}$$

- EM and QCD dipole operators:

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2}(\bar{s}\sigma^{\mu\nu} P_R b)F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{g_s}{(4\pi)^2} m_b(\bar{s}\sigma^{\mu\nu} P_R T^a b)G^a_{\mu\nu} \tag{18}$$

- The semileptonic operators:

$$\mathcal{O}_9 = \frac{\alpha_{em}}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \quad \mathcal{O}_{10} = \frac{\alpha_{em}}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell). \tag{19}$$

$\mathcal{O}'_{9,10}$ with $P_L \to P_R$ in quark currents

Angular Observables in $B \to K^*\ell\ell$

- Complete information about the decay in full four-fold angular distributions:

$$\begin{aligned}\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} &= \frac{3}{8\pi} J(q^2, \cos\theta_\ell, \cos\theta_K, \phi),\\ \frac{d^4\bar{\Gamma}}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} &= \frac{3}{8\pi} \bar{J}(q^2, \cos\theta_\ell, \cos\theta_K, \phi),\end{aligned} \tag{20}$$

with

$$\begin{aligned}J(q^2, \theta_\ell, \theta_K, \phi) &= J_1^s \sin^2\theta_K + J_1^c \cos^2\theta_K + (J_2^s \sin^2\theta_K + J_2^c \cos^2\theta_K)\cos 2\theta_\ell\\ &+ J_3 \sin^2\theta_\ell \sin^2\theta_K \cos 2\phi + J_4 \sin 2\theta_\ell \sin 2\theta_K \cos\phi\\ &+ J_5 \sin\theta_\ell \sin 2\theta_K \cos\phi + J_6 \cos\theta_\ell \sin^2\theta_K\\ &+ J_7 \sin\theta_\ell \sin 2\theta_K \sin\phi + J_8 \sin 2\theta_\ell \sin 2\theta_K \sin\phi\\ &+ J_9 \sin^2\theta_\ell \sin^2\theta_K \sin 2\phi.\end{aligned}$$

- The LHCb Collaboration measured the CP-averaged ratios[2]

$$S_i \equiv \frac{J_i + \bar{J}_i}{d\Gamma/dq^2 + d\bar{\Gamma}/dq^2}. \tag{21}$$

[2]Note the different convention $F_L = F_L^{\mathrm{LHCb}}$, $S_{3,5,7,9} = \frac{3}{4} S_{3,5,7,9}^{\mathrm{LHCb}}$, $S_{4,8} = -\frac{3}{4} S_{4,8}^{\mathrm{LHCb}}$, $S_5 = \frac{3}{4} S_5^{\mathrm{LHCb}}$, $A_{FB} = -A_{FB}^{\mathrm{LHCb}}$