False Vacuum Decay in Scalar Field Theories

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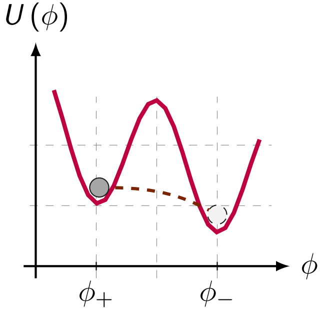
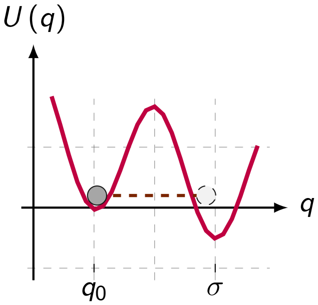
Outline: Introduction

1. Tunnel Decay in Field Theory
2. False Vacuum Decay in the SM
3. Multiple False
   Vacuum Decay in an Effective QCD



Tunneling in One Dimension



Decay Rate

$$\frac{\Gamma}{V} = Ae^{-B}\left[1 + O(\hbar)\right]$$

$$B = 2\int_{q_0}^{\sigma} dq\,(2U)^{1/2}$$

Tunneling in One Dimension

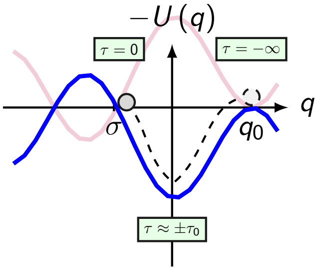
The Bounce

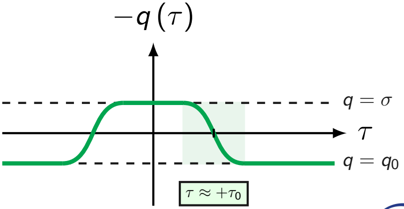
$$\int_{q_0}^{\sigma} dt\,(2U)^{1/2} = \int_{-\infty}^{0} d\tau L_E$$

$$\left.\frac{dq}{d\tau}\right|_{\tau_\sigma=0} = 0$$

$$\lim_{\tau\to-\infty} q = q_0$$

$$B = \int_{-\infty}^{\infty} d\tau L_E \equiv S_E .$$

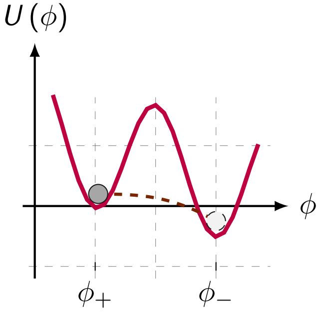




Tunneling in Scalar Field Theory

The Action

$$S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + U\right]$$



$$\lim_{\tau\to\pm\infty} \phi(\tau,\vec{x}) = \phi_+$$

$$\frac{\partial\phi}{\partial\tau}(0,\vec{x}) = 0$$

$$\lim_{|\vec{x}|\to\infty} \phi(\tau,x) = \phi_+$$

Tunneling in Scalar Field Theory

The Action and EoM

$$\rho = \left(\tau^2 + |\vec{x}|^2\right)^{1/2}$$

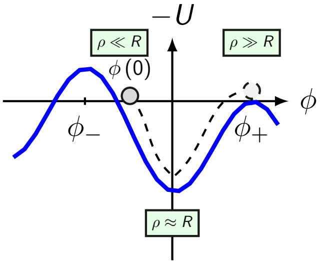
$$S_E(\phi) = 2\pi^2 \int_0^\infty \rho^3 d\rho \left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2 + U\right]$$

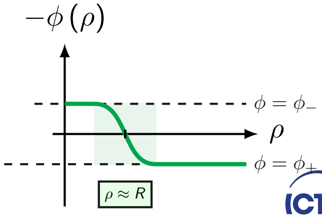
$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = U'(\phi),$$

Boundary Conditions

$$\lim_{\rho\to\infty} \phi(\rho) = \phi_m.$$

$$\left.\frac{d\phi}{d\rho}\right|_0 = 0.$$

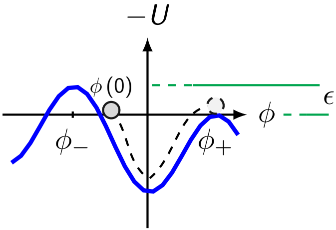




The Coleman's Thin Wall Approximation

EoM

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = U'(\phi),$$



In the Wall

$$\frac{d^2\phi}{d\rho^2} = \frac{dU_0}{d\phi}$$

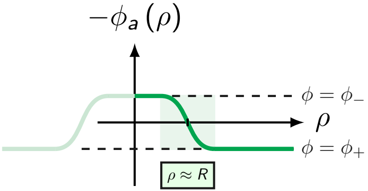
$$\begin{aligned} S_E &= 2\pi^2 R^3 \int_0^\infty d\rho \left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2 + U_0\right] \\ &= 2\pi^2 R^3 S_1 \end{aligned}$$

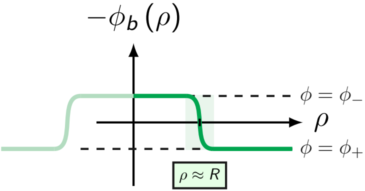
The Coleman's Thin Wall Approximation

In the Thin Wall Approximation

$$\begin{aligned}\phi &= \phi_-, \quad \rho \ll R \quad \Rightarrow B_{inside} = -\frac{\pi^2}{2} R^4 \epsilon \\ &= \phi_1 (\rho - R), \quad \rho \approx R \quad \Rightarrow B_{wall} \equiv 2\pi^2 R^3 S_1 \\ &= \phi_+, \quad \rho \gg R \quad \Rightarrow B_{outside} = 0\end{aligned}$$

$$\frac{dS_E}{dR} = 0 = -2\pi^2 R^3 \epsilon + 6\pi^2 R^2 S_1 \quad \Rightarrow \quad R = \frac{3S_1}{\epsilon}$$





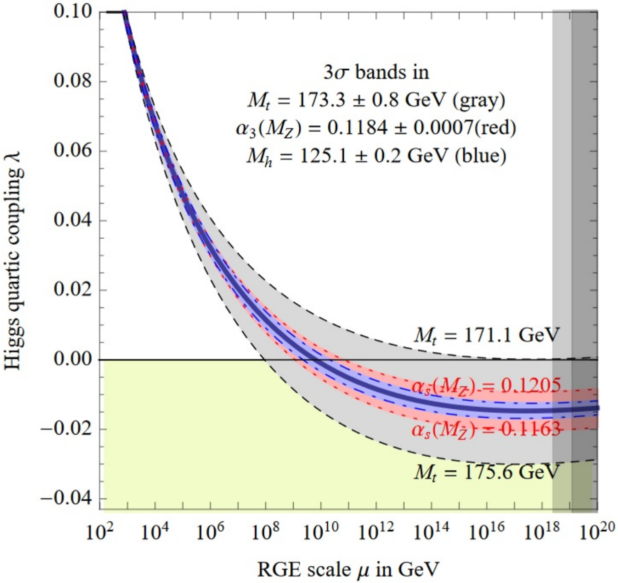
False Vacuum Decay in the SM

$$U = -\mu^2|\Phi^2| + \lambda|\Phi^4| = \frac{1}{2}m^2 h^2 + \sqrt{m\lambda}\, h^3 + \frac{1}{4}\lambda h^4 + ...$$

The Scalar Effective Potential in SM

$$\mathbf{U}\left(h \gg v\right) \cong \frac{1}{4}\lambda_{eff}\left(h\right) h^4$$

False Vacuum Decay in the SM



[arXiv:1307.3536 [hep-ph]]

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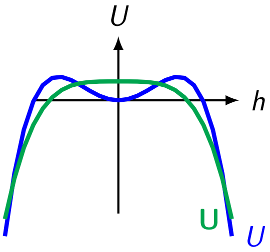
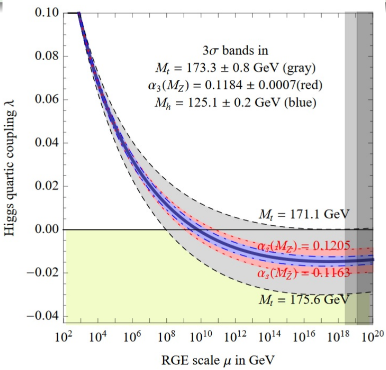




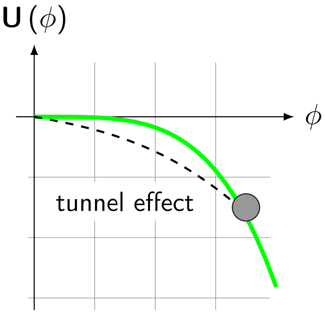
[arXiv:1307.3536 [hep-ph]]

Tunnel Without Barrier

$$\mathbf{U}(\phi) = \frac{\lambda}{4}\phi^4$$

with $\lambda < 0$

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = \mathbf{U}'(\phi) = \lambda\phi^3$$



$$S_E = 2\pi^2 \int_0^\infty \rho^3 d\rho \left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2 + \mathbf{U}\right] = \frac{8\pi^2}{3|\lambda|}$$

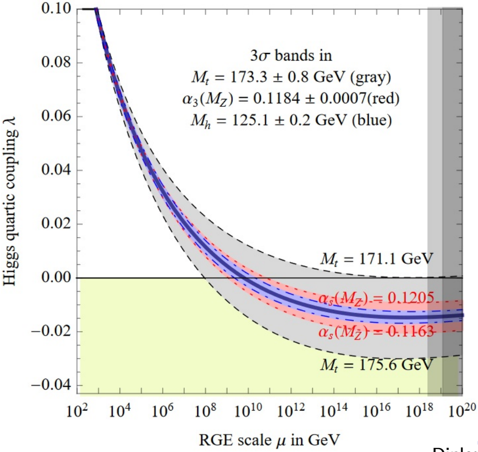
The Action is Scale Invariant

$$\rho \to \alpha\,\rho \qquad \phi \to \frac{1}{\alpha}\,\phi$$

Decay Rate of the False Vacuum in the SM

$$\Gamma \simeq \frac{V_U}{R^4} e^{-\frac{8\pi^2}{3|\lambda|}}$$

The most probable energy of decay is approximately $\mu \simeq 10^{16}\,GeV$



Decay Rate of the False Vacuum in the SM

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$$\Gamma \simeq \left(10^{25} eV\right)^4 \left(8.10^{101} eV^{-3}\right) e^{-\frac{8\pi^2}{3|-0.02|}} \simeq 10^{-370} eV$$

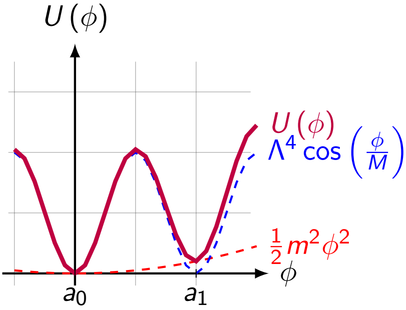
$$t \equiv \frac{1}{\Gamma} \simeq 10^{347} \; years$$

Multiple Vacua Potential

Effective Potential

$$U(\phi) = \frac{1}{2}m^2\phi^2 - \Lambda^4 \cos\left(\frac{\phi}{M}\right)$$

we assume that $m^2 \ll \frac{\Lambda^4}{M^2}$

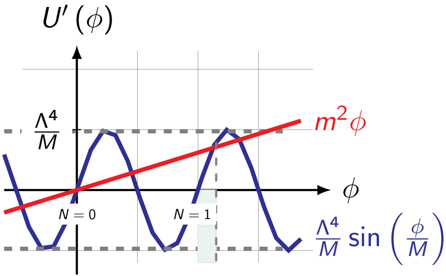


Multiple Vacua Potential

First Derivative of the E.P.

$$U'(\phi_m) = m^2\phi_m + \frac{\Lambda^4}{M}\sin\left(\frac{\phi_m}{M}\right) = 0$$

we will assume that $m^2 \ll \frac{\Lambda^4}{M^2}$

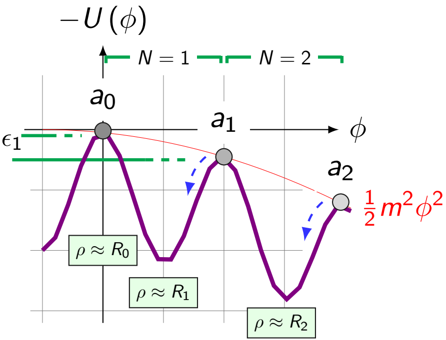


$$m^2 \ll \frac{\Lambda^4}{M^2}$$

$$N \leq \frac{1}{2\pi}\left(\frac{\Lambda^2}{Mm}\right)^2.$$

Vacua Decay in The Thin-Wall Approximation

$$m^2 \ll \frac{\Lambda^4}{M^2}$$



$$S_N = \int_{a_{N-1}}^{a_N} d\phi \left[2U(\phi)\right]^{1/2}$$

$$S_N \simeq S_{N-1} \simeq \ldots \simeq S_1$$

The Euclidean Action

$$\hat{S}_N \simeq -\frac{1}{2}\pi^2 \epsilon_N R_N^4 + 2\pi^2 S_1 R_N^3$$

$$\epsilon_N = \frac{1}{2} m^2 \left(a_N^2 - a_{N-1}^2\right).$$

In a Specific QCD Effective Potential

Effective Potential

$$U = \frac{1}{2} m_{\eta'}^2 \eta'^2 - m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{\eta'}{f_\pi}\right)$$

$$m_{\eta'} = m_{\eta'_0}\sqrt{\frac{3}{N_c}} \qquad f_\pi = f_{\pi_0}\sqrt{\frac{N_c}{3}}.$$

$$\epsilon = \frac{1}{2} m_{\eta'}^2 \left(a_N^2 - a_{N-1}^2\right) \simeq 4\pi^2 f_\pi^2 m_{\eta'}^2 \left(N - \frac{\theta}{2\pi}\right)$$

Decay rate from N to $N-1$

$$\frac{\Gamma_N}{V} \simeq e^{-\hat{S}_N}$$

$$\hat{S}_N \simeq \frac{27}{2\pi^4}\left(\frac{8 m_\pi^2 f_\pi}{m_{\eta'}^3}\right)^2 \frac{1}{\left(N - \frac{\theta}{2\pi}\right)^3}.$$

In a Specific QCD Effective Potential

$$N \leq \frac{1}{2\pi}\left(\frac{m_\pi}{m_{\eta'}}\right)^2 + \frac{\theta}{2\pi}$$

In the large N_c limit

$$\epsilon \simeq 4\pi^2 f_{\pi_0}^2 m_{\eta_0'}^2 \left(N - \frac{\theta}{2\pi}\right) \qquad \hat{S}_N \simeq \frac{27}{2\pi^4}\left(\frac{8 m_\pi^2 f_{\pi_0}}{9 m_{\eta_0'}^3}\right)^2 \frac{N_c^4}{\left(N - \frac{\theta}{2\pi}\right)^3}.$$

$$\Gamma_N \sim e^{-kN_c^4/N^3}$$

For $N = O(N_c)$

$$\Gamma_{O(N_c)} \sim e^{-N_c}$$

For $N = O(1)$

$$\Gamma_{O(1)} \sim e^{-N_c^4} \ll 1$$

Summary

- We reviewed Coleman's results, which makes possible the computation of the vacuum decay in quantum field theory
- Vacuum decay in the Higgs potential of the Standard Model. We found that the most probable time of decay is of order 10^{347} years, which is larger than the age of the universe.
- Vacuum decay in a QCD effective potential, in the $N_c \gg 1$ limit. We find that the higher vacua are more unstable and that the transition rate is suppressed exponentially with the number of colors.

Hvala