Killing the ρ, the $K^\star$, and their ugly cousins

Towards a coherent approach for B decays to unstable particles

Javier Virto

Universität Bern

Brda - Slovenia – October 20, 2016

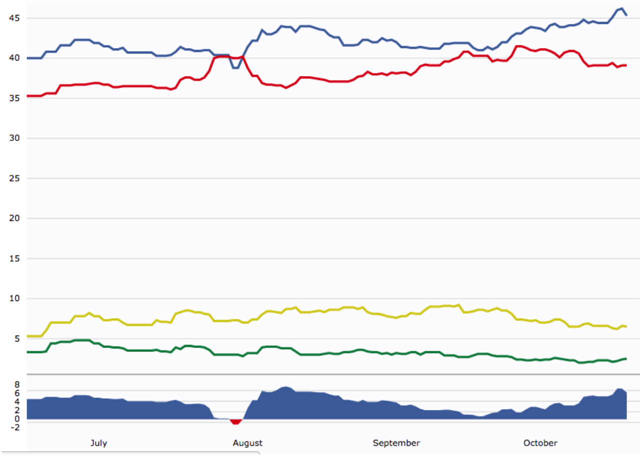
Based on collaborations with S. Kränkl, T. Mannel, A. Khodjamirian, S. Cheng, T. Huber and K. Vos

b UNIVERSITÄT BERN

It is great to be in Slovenia

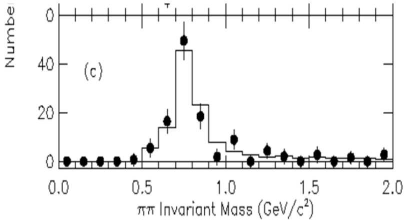
The home country of the next first lady...

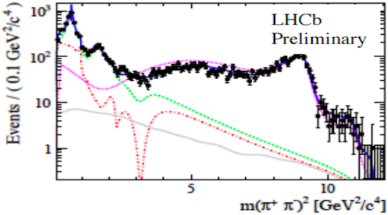
Or maybe not ...



:: What is a ρ meson?

▷ **Experimentalist version** : Some bump in a $\pi\pi$ distribution





Left: the current source for $B^- \to D^0 \rho^-$ (CLEO). Right: $B \to D\pi^+\pi^-$ (LHCb).

▷ **Theorist version** : A pole in some correlation function

$$\mathrm{Im}\left[\int d^4x\ e^{iq\cdot x}\langle 0|\,T\{j^\mu(x), j_\mu(0)\}|0\rangle\right] \sim \delta(q^2 - m_\rho^2)\cdot f_\rho^2 + \cdots$$

▷ Be not surprised that: **Experimentalist version** $\neq$ **Theorist version**

:: Why do we care about this?

- Measurement of V_{ub}

$$\begin{pmatrix} B \to \rho\,\ell\nu \\ B_s \to K^\star \ell\nu \end{pmatrix} \longrightarrow \text{ Really are } \begin{pmatrix} B \to \pi\pi\,\ell\nu \\ B_s \to K\pi\,\ell\nu \end{pmatrix}$$

- Rare penguin decays (NP)

$$\begin{pmatrix} B \to \rho\,\ell\ell \\ B \to K^\star \ell\ell \end{pmatrix} \longrightarrow \text{ Really are } \begin{pmatrix} B \to \pi\pi\,\ell\ell \\ B \to K\pi\,\ell\ell \end{pmatrix}$$

- quasi-two-body decays (α, CP violation, NP ...)

$$\begin{pmatrix} B \to \rho\pi \\ B \to K^\star\pi \\ \cdots \end{pmatrix} \longrightarrow \text{ Really are } \begin{pmatrix} B \to \pi\pi\pi \\ B \to K\pi\pi \\ \cdots \end{pmatrix}$$

- Huge experimental programs for these modes at LHCb and Belle-2
  Huge data sets will require theory precision.

:: Main theory objects

$B \to \rho$ form factors	$\longleftarrow \cdots \longrightarrow$	$B \to \pi\pi$ form factors
$\langle \rho \| \bar{q}(x)\Gamma b(0) \| \bar{B} \rangle$		$\langle \pi\pi \| \bar{q}(x)\Gamma b(0) \| \bar{B} \rangle$
ρ-LCDAs...	$\longleftarrow \cdots \longrightarrow$	2π-LCDAs...
$\langle \rho \| \bar{q}(x)\Gamma q(0) \| 0 \rangle_{x^2 \to 0}$		$\langle \pi\pi \| \bar{q}(x)\Gamma q(0) \| 0 \rangle_{x^2 \to 0}$
...and its normalization f_ρ	$\longleftarrow \cdots \longrightarrow$	...and its normalization $F_\pi(s)$

OUTLINE of the talk: Three examples:

- $B \to \rho$ and $B \to \pi\pi$ form factors from B-meson LCSRs
- $B \to \pi\pi$ form factors from 2π LCDAs
- Quasi-two-body non-leptonic decays

:: QCD Sum Rules: crash course

- Imagine you have some correlation function

$$\Pi(q^2,\dots) = \int d^4x\ e^{iq\cdot x}\,\langle\alpha|\,T\{j_1(x),j_2(0)\}|\beta\rangle$$

with (only) a cut for real $q^2 > s_{th}$, and calculable via some OPE at $q^2 = \bar{q}^2$.

- One can write a dispersion relation:

$$\Pi_{\rm OPE}(\bar{q}^2,\dots) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds\,\frac{{\rm Im}\Pi(s,\dots)}{s-\bar{q}^2}$$

- The L.H.S. can be calculated perturbatively (by assumption) in conjunction with a power expansion, and the R.H.S. is given by unitarity:

$$2{\rm Im}\Pi(s) = (2\pi)\delta(s-m_\lambda^2)\langle\alpha|j_1|\lambda\rangle\langle\lambda|j_2|\beta\rangle + \text{higher states}$$

- Finally, a Borel Transformation $\bar{q}^2 \to M^2$ takes care of possible subtractions, convergence of the OPE and suppression of higher states:

$$\boxed{\Pi_{\rm OPE}(M^2,\dots) = \langle\alpha|j_1|\lambda\rangle\langle\lambda|j_2|\beta\rangle e^{-m_\lambda^2/M^2}}$$

:: $B \to \rho$ form factors from B-meson LCSRs

Khodjamirian, Mannel, Offen '06

▷ **Correlation function**

$$F_\mu(k,q) = i \int d^4x e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0) i m_b \gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

▷ **Unitarity relation**

$$\begin{aligned} 2\mathrm{Im}F_\mu(k,q) &= \sum_\lambda (2\pi)\delta(k^2 - m_\rho^2) \underbrace{\langle 0|\bar{d}\gamma_\mu u|\rho_\lambda(k)\rangle}_{m_\rho f_\rho\, \varepsilon(\lambda)_\mu} \underbrace{\langle \rho_\lambda(k)|\bar{u} i m_b \gamma_5 b|\bar{B}^0(q+k)\rangle}_{(\varepsilon(\lambda)^\star \cdot q)\ A_0^{B\rho}(q^2)} + \cdots \\ &= q_\mu\ 4\pi\, m_\rho\, f_\rho\, A_0^{B\rho}(q^2) + \cdots \end{aligned}$$

▷ **Dispersion relation + LCOPE + Borel + duality**

$$2 m_\rho\, f_\rho\, A_0^{B\rho}(q^2) e^{-m_\rho^2/M^2} = F_{OPE}(M^2, q^2)$$

:: $B \to \pi\pi$ form factors from B-meson LCSRs

Cheng, Khodjamirian, JV '16?

▷ **Correlation function**

$$F_\mu(k,q) = i\int d^4x e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), m_b\bar{u}(0)\gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

▷ **Unitarity relation**

$$\begin{aligned} 2\mathrm{Im}F_\mu(k,q) &= m_b\int d\tau_{2\pi}\,\underbrace{\langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle}_{F_\pi^\star(s)}\underbrace{\langle \pi(k_1)\pi(k_2)|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle}_{F_t(s,q^2,\cos\theta_\pi)} + \cdots \\ &= q_\mu\frac{s\sqrt{q^2}\beta_\pi(s)^2}{4\sqrt{6}\pi\sqrt{\lambda}}\,F_\pi^\star(s)\;F_t^{(\ell=1)}(s,q^2) + \cdots \end{aligned}$$

Corollary: $F_\pi^\star(s)\;F_t^{(\ell=1)}(s,q^2)$ is real for all $s < 16m_\pi^2 \Rightarrow$

$$\text{Phase}(F^{B\to\pi\pi}) = \text{Phase(pion form factor)}$$

Important for CP violation!!! [See also Kang, Kubis, Hanhart, Meissner '13]

:: $B \to \pi\pi$ form factors from B-meson LCSRs

Cheng, Khodjamirian, JV '16?

▷ **Dispersion relation + LCOPE + Borel + duality**

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\ e^{-s/M^2}\ \frac{s\ \sqrt{q^2}\ [\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}}\ F_\pi^\star(s)\ F_t^{(1)}(s,q^2) = F_{OPE}(M^2,q^2)$$

▷ **ρ-dominance + zero-width limit:**

$$F_\pi^\star(s) \simeq \frac{f_\rho g_{\rho\pi\pi} m_\rho/\sqrt{2}}{m_\rho^2 - s + i\sqrt{2}\Gamma_\rho(s)} \quad , \quad F_t^{(1)}(s,q^2) \simeq -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3q^2}}\ \frac{m_\rho g_{\rho\pi\pi} A_0^{B\rho}(q^2)}{m_\rho^2 - s - i\sqrt{2}\Gamma_\rho(s)}$$

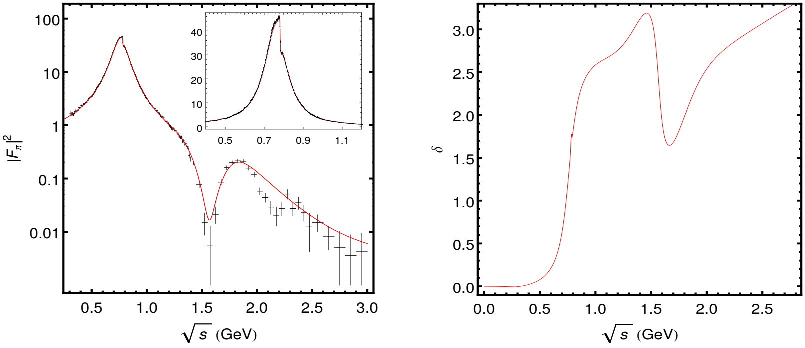
$$LHS = 2f_\rho m_\rho A_0^{B\rho}(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds\ e^{-s/M^2} \underbrace{\left[\frac{\sqrt{s}\ \Gamma_\rho(s)/\pi}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)}\right]}_{\xrightarrow{\Gamma_\rho\to 0}\ \delta(s-m_\rho^2)} \xrightarrow{\Gamma_\rho\to 0} 2f_\rho m_\rho A_0^{B\rho}(q^2)\ e^{-m_\rho^2/M^2}$$

ρ sum rule ✓

:: $B \to \pi\pi$ form factors from B-meson LCSRs

Cheng, Khodjamirian, JV '16?

The main input is the pion form factor:



Source: $e^+e^- \to \pi\pi(\gamma)$ [Babar] and $\tau \to \pi\pi\ell\nu$ [Belle].

:: $B \to \pi\pi$ form factors from 2π-LCDAs

Hambrock, Khodjamirian, 2015; Cheng, Khodjamirian, JV w.i.p

- **Correlation function**

$$\Pi^5(p^2, k^2, q^2, q\cdot\bar{k}) = i\int d^4x\, e^{iq\cdot x}\langle \pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{\bar{u}(x)im_b\gamma_5 b(x), \bar{b}(0)im_b\gamma_5 d(0)\}|0\rangle$$

- **Unitarity relation**

$$2\mathrm{Im}\Pi^5 = (2\pi)\delta(p^2 - m_B^2)\underbrace{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}im_b\gamma_5 b|\bar{B}(p)\rangle}_{\sqrt{q^2}F_t(q^2,k^2,q\cdot k)}\underbrace{\langle \bar{B}(p)|\bar{b}im_b\gamma_5 d|0\rangle}_{m_B^2 f_B} + \cdots$$

$$= (2\pi)\,\delta(p^2 - m_B^2)\, m_B^2\, f_B\,\sqrt{q^2}\, F_t(q^2, k^2, q\cdot k) + \cdots$$

- **Dispersion relation + LCOPE + Borel + duality**

$$m_B^2\, f_B\,\sqrt{q^2}\, F_t(q^2, k^2, q\cdot k)\, e^{-m_B^2/M^2} = \Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k})$$

:: $B \to \pi\pi$ form factors from 2π-LCDAs

Hambrock, Khodjamirian, 2015; Cheng, Khodjamirian, JV w.i.p

- In this case:

$$\Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k}) = \frac{m_b^2}{\sqrt{2}} \int_{u_0}^1 \frac{du}{u^2}\, e^{-s(u)/M^2} \left(m_b^2 - q^2 + u^2 k^2\right) \Phi_\parallel(u, q\cdot\bar{k}, k^2)$$

- Where the 2π LCDA is defined as

$$\Phi^q_\parallel(z,\zeta,s) = \int \frac{dx^-}{2\pi}\, e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- The 2π LCDA is normalized to the pion form factor:

$$\int dz\, \Phi_\parallel(z,\zeta,s) = (2\zeta - 1)\, F_\pi(s)$$

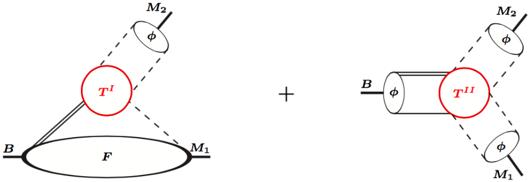
but for the sum rule we need higher moments.

- Narrow-ρ dominance on $\Phi_\parallel$ leads to $B \to \rho$ form factor from ρ-LCDA. ✓

:: Two-body decays

To leading power in the heavy-quark expansion Beneke, Buchalla, Neubert, Sachrajda '99

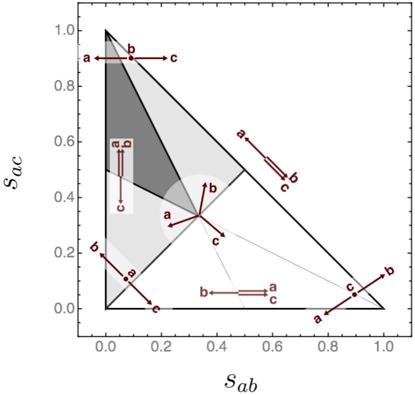
$$\langle M_1 M_2|\mathcal{O}_i|B\rangle = F^{BM_1}\int du\, T_i^I(u)\phi_{M_2}(u) + \int d\omega\, du\, dv\, T_i^{II}(\omega,u,v)\phi_B(\omega)\phi_{M_1}(u)\phi_{M_2}(v)$$

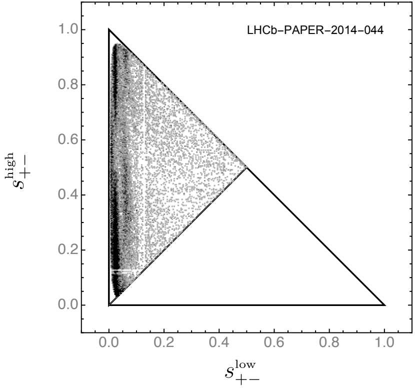


- Vertex corrections: $T^I(u) = 1 + \mathcal{O}(\alpha_s/\pi)$
- Spectator scattering: $T^{II}(\omega,u,v) = \mathcal{O}(\alpha_s)$
- This applies for example to $B \to \rho\pi$.

:: Three-body decays – kinematics

- ▷ $\bar{B} \to M_a(p_a) M_b(p_b) M_c(p_c)$
- ▷ Two independent invariants, *e.g.* $s_{ab} = \frac{(p_a+p_b)^2}{m_B^2}$ and $s_{ac} = \frac{(p_a+p_c)^2}{m_B^2}$





:: $B \to \pi\rho$ vs. $B \to \pi\pi\pi$

This is **always** an improvement w.r.t. quasi-two-body decays:

$$\mathcal{A}(B^- \to \pi^-[\pi^+\pi^-]) = F^{B\to\pi}\ T_1 \star \phi_{\pi\pi} + F^{B\to\pi\pi}\ T_2 \star \phi_\pi$$

$$\Big\downarrow \quad \rho \text{ dominance} + \text{zero-width limit}$$

$$\mathcal{A}(B^- \to \pi^-\rho) = F^{B\to\pi}\ T_1 \star \phi_\rho + F^{B\to\rho}\ T_2 \star \phi_\pi$$

This limit can be checked analytically.

- ▷ Factorization is at the same level of theoretical rigour for quasi-two-body and 3-body.
- ▷ Any model for $\phi_{\pi\pi}$ and $F^{B\to\pi\pi}$ satisfying axiomatic constraints and compatible with data (e.g. $e^+e^- \to \pi\pi$) replaces any notion of "ρ".

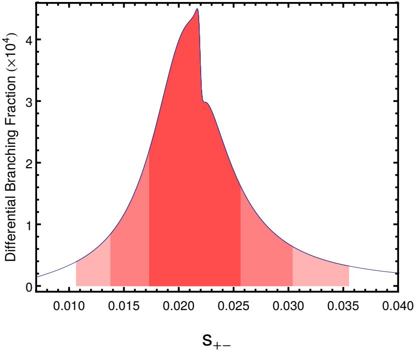
:: $B \to \pi\rho$ vs. $B \to \pi\pi\pi$

⋆ Leading order amplitude: Krankl, Mannel, JV '15

$$\mathcal{A}|_{s_{+-}\ll 1} = \frac{G_F}{\sqrt{2}}\Big[4m_B^2 f_0(s_{+-})(2\zeta-1)F_\pi(s_{+-})(a_2+a_4) + f_\pi m_\pi(a_1-a_4)F_t(\zeta,s_{+-})\Big]$$

⋆ Integrating around the ρ:

$$BR(B^- \to \rho\pi^-) \simeq \int_0^1 ds_{++}\int_{s_\rho^-}^{s_\rho^+} ds_{+-}\,\frac{\tau_B\, m_B|\mathcal{A}|^2}{32(2\pi)^3}$$



with $s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2/m_B^2$

$BR(B^+ \to \rho\pi^+) \simeq 9.4\cdot 10^{-6}$ $(n=0.5)$

$BR(B^+ \to \rho\pi^+) \simeq 12.8\cdot 10^{-6}$ $(n=1)$

$BR(B^+ \to \rho\pi^+) \simeq 14.1\cdot 10^{-6}$ $(n=1.5)$

$BR(B^+ \to \rho\pi^+)_{\rm EXP} = (8.3\pm 1.2)\cdot 10^{-6}$

$BR(B^+ \to \rho\pi^+)_{\rm QCDF} = (11.9^{+7.8}_{-6.1})\cdot 10^{-6}$

:: Summary

- B-decays to unstable particles can be described model-independently in terms of their underlying multi-body decays.
- Need better knowledge of $(\pi\pi)$, $(K\pi)$, ... light-cone distributions, including the local normalizations, and in different partial waves.
- Multi-body form factors can be calculated using LCSRs (not lattice!!)
- Very important in view of future luminosities at LHCb and Belle-2.
- This is just a glimpse.