Simulating P_c^+ channel in the lattice QCD

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Overview

Motivation

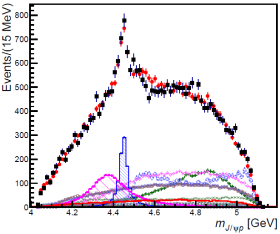
- Most hadrons are hadronic resonances. Some of them decay strongly and can be studied on the lattice.

$$H_1 + H_2 \rightarrow R \rightarrow H_1 + H2$$

- Study of scattering on the lattice is similar to scattering in experiment.
- In 2015 charmed pentaquark state P_c^+, decaying into $N + J/\psi$ was discovered by LHCb (LHCb; *PRL*, 2015(115),072001).

$$N + J/\psi \rightarrow P_c^+ \rightarrow N + J/\psi$$

- Two states were observed:
  - lower state with $J = \frac{3}{2}$, mass $m_{P_c^+} = 4380 \pm 8\text{MeV}$ and width $\Gamma = 205 \pm 18\text{MeV}$
  - upper state with $J = \frac{5}{2}$, mass $m_{P_c^+} = 4449{,}8 \pm 1{,}7\text{MeV}$ and width $\Gamma = 39 \pm 5\text{MeV}$
  - states have oposite parity. It is not clear which state is positive and which negative under parity transformation.
- Study of this channels was not performed on the lattice yet.



Basics of LQCD

- Discretization of QCD equations in Euclidian space
- Periodic boundary conditions in space
- Quarks (fermionic fields) live in corners of lattice
- connected are with gluons (quants of gauge field)

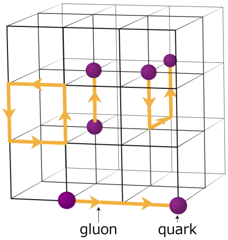


Figure: Magenta dots (quarks, ψ) are connected with yellow lines (gluon A_μ)

Path integral in LQCD

Calculation of physically observable quantity

1. Expected value of operator
2. Feynman path integral
3. Correlation function (matrix)

Path integral

$$\langle\Omega|O_i(t)O_j^\dagger(0)|\Omega\rangle = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu O_i(t)O_j^\dagger(0)\,\exp^{-S_G(A_\mu)}\exp^{-S_F(\psi,\bar{\psi},A_\mu)}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \exp^{-S_G(A_\mu)}\exp^{-S_F(\psi,\bar{\psi},A_\mu)}}$$

Correlation function

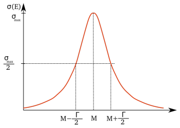
$C_{ij} = \langle\Omega|O_i(t)O_j^\dagger(0)|\Omega\rangle$, $O = H_1 H_2$, where H is a hadron operator

$$H_1 + H_2 \longleftrightarrow R$$

- Information about hadronic resonances are calculated with simulation of several operators for initial and final state

m_R and Γ determination

1. n interpolator O_i in simulation
2. Result of simulation: $n \times n$ matrix C
3. Energy spectral decomposition: $E_n(L)$
4. Luscher's Equation: $E \rightarrow \delta(E)$
5. Scattering matrix: $T(E)$, $(T(E) \propto \sigma)$
6. Determination of : m_R and Γ

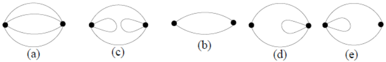


Usefull relations

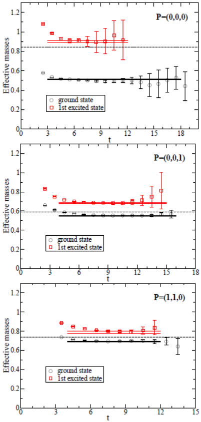
- **Discrete value of momentum:** $p = \vec{N}\frac{2\pi}{L}, \vec{N} \in \mathbf{N}^3$
- **Energy of two hadrons in a box:** $E(L) = \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2} + \Delta E$
- **Correlation function:** $C_{ij}(t) = \langle \Omega | O_i(t) O_j^\dagger(0) | \Omega \rangle$
- **Energy spectral decomposition:** $C_{ij}(t) = \sum_n \langle \Omega | O_i(t) | n \rangle \langle n | O_j^\dagger(0) | \Omega \rangle \exp^{-E_n t}$
- **Scattering matrix:** $S(E) = \exp^{2i\delta(E)}$, $S(E) = 1 + 2iT(E)$
- **Breit-Wigner resonances:** $T(E) = \frac{-E\Gamma}{E^2 - m_R^2 + iE\Gamma}$, $\Gamma(E) = g^2 \frac{p^{2l+1}}{E^2}$

Example: ρ resonance on the lattice

Older results but neat example. (S. Prelovsek et al.: *PRD* 84, (2011),054503)



$$O = \bar{q}q, (\bar{q}q)(\bar{q}q)$$



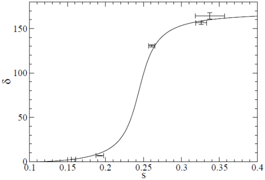
$E \to \delta(E)$

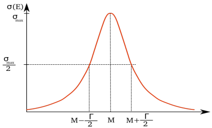
Luscher's equation

$$T(E) = \frac{1}{\cot(\delta(E)) - i}$$

$$T(E) = \frac{-E\Gamma}{E^2 - m_R^2 + iE\Gamma}$$

$$\Gamma(E) = g_{\rho\pi\pi}^2 \frac{p^{2l+1}}{6\pi E^2}$$

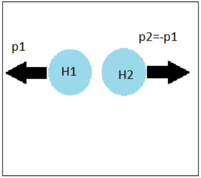


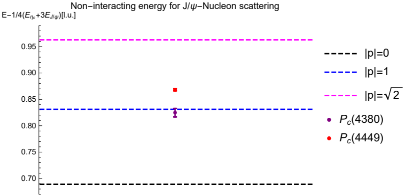


ρ meson	Mass [MeV]	$g_{\rho\pi\pi}$
Latt (m_π = 266MeV)	$772 \pm 6 \pm 8$	$5{,}61 \pm 0{,}12$
Experiment	775	5,97

Strong decay of P_c^+

- Pentaquark consists of 4 quarks (u, u, d, c) and 1 antiquark ($\bar{c}$)
- Simulation are made in approximation of 1 channel scattering for $J/\psi - p$
- Simulations are made for total momentum $\vec{P} = \vec{p}_{H_1} + \vec{p}_{H_2} = 0$
- Resonance is possible to see in cases where momentum of single particle $|p_{H_i}| \geq 1$
- It should be sufficient to study scattering until $|p_{H_i}| = \sqrt{2}$, we should be able to see both P_c^+ states





Dispersion relation: $E \approx \sqrt{m^2 + p^2}$

Introduction to single hadron results

- Properties of used lattice

$N^3 \times N_T$	a[fm]	L[fm]	#config	m_π[MeV]
$16^3 \times 32$	0.1239(13)	1.98	280	266(3)

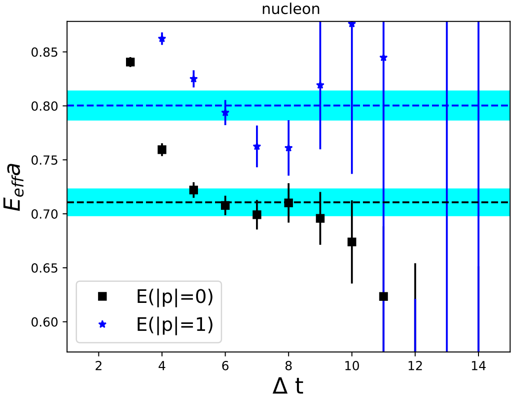
- On used lattice both used hadrons were simulated before by SP, CBL and their groups
- This is a good test of our code

Expected energies of observed hadrons on used lattice measured by other authors

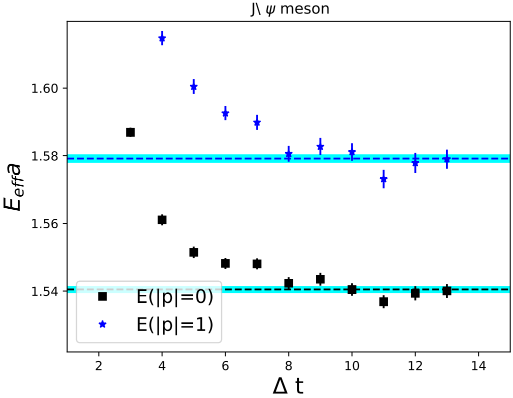
state	value (ma)	error	fit range
nucleon (ground state)	0.672	0.004	[6,12]
J/ψ (ground state)	1.54171	0.00043	[3,27]

Nucleon N

- Results for nucleon with momentum $|p| = 0$ and $|p| = 1$
- Simulations were made for 6 operators (3 for $|p| = 0$ and 3 for $|p| = 1$),4 of them were used in analysis
- It is difficult to successfully determine where plateau is



J/ψ meson



- Results for J/ψ meson with momentum $|p| = 0$ and $|p| = 1$
- Obtained energies agree perfect with expected values:
  $E(|p| = 0) = 1.542$ and
  $E(|p| = 1) = 1.579$

Combining single hadron correlators

- Irreps correspond to determined continuum angular momentum J
- Single hadron correlators were calculated separately for all combinations of momentum and polarizations
- Combination of single hadron correlators in correlator corresponding to desired quantum numbers in each irrep

J	irrep
0	A_1
$\frac{1}{2}$	G_1
1	T_1
$\frac{3}{2}$	H
2	$E \oplus T_2$
$\frac{5}{2}$	$H \oplus G_2$
3	$A_2 \oplus T_1 \oplus T_2$
$\frac{7}{2}$	$G_1 \oplus H \oplus G_2$
4	$A_1 \oplus E \oplus T_1 \oplus T_2$

All explicit expressions for $H_1(p)H_2(-p)$ operators with desired quantum numbers in desired irrep are given in our paper (S. Prelovsek, U.S., C.B. Lang ; *JHEP* 2017(1), 129.).

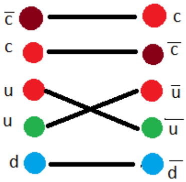
Example: Scattering in P_c^+ pentaquark candidate channel: for irrep H^- and $J=\frac{3}{2}, S=\frac{3}{2}, L=0$ and $|p|=0$

Anihilation operator for this example is:

$$O^{H^-, r=1}_{J=\frac{3}{2}, S=\frac{3}{2}, L=0}(0) = N_{\frac{1}{2}}(0)\left(V_x(0) - iV_y(0)\right)$$

Creation operator:

$$\bar{O}^{H^-, r=1}_{J=\frac{3}{2}, S=\frac{3}{2}, L=0}(0) = N_{\frac{1}{2}}(0)\left(V_x(0) + iV_y(0)\right)$$



Therefore correlation function for this quantum numbers in H^- irrep is sum over product of single hadron correlators:

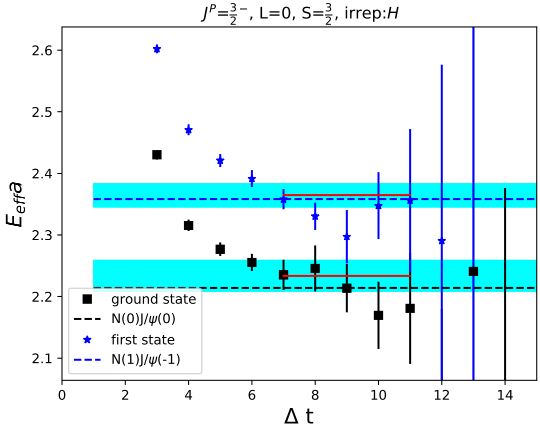
$$C^{VN;H^-}_{J=\frac{3}{2}, S=\frac{3}{2}, L=0}(|p|=0) = \langle \Omega | O^{H^-}_{J=\frac{3}{2}, S=\frac{3}{2}, L=0} \bar{O}^{H^-}_{J=\frac{3}{2}, S=\frac{3}{2}, L=0} | \Omega \rangle =$$

$$C^N_{\frac{1}{2}\to\frac{1}{2}} C^V_{x\to x} - iC^N_{\frac{1}{2}\to\frac{1}{2}} C^V_{x\to y} + iC^N_{\frac{1}{2}\to\frac{1}{2}} C^V_{y\to x} + C^N_{\frac{1}{2}\to\frac{1}{2}} C^V_{y\to y}$$

where:

$$C^H_{pol_{src}\to pol_{snk}} = \langle \Omega | H_{pol_{snk}} \bar{H}_{pol_{src}} | \Omega \rangle$$

Results for irrep H with parity $P = -1$ and quantum numbers $J = \frac{3}{2}, S = \frac{3}{2}, L = 0$

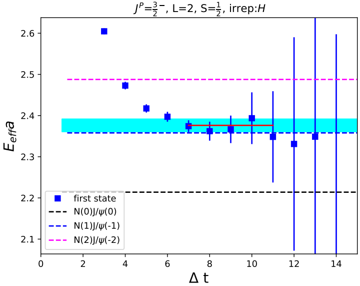


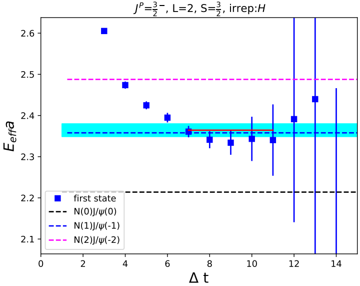
- We are looking for energy shift
- This would be sign that we have some interaction
- No signs for interaction can be seen for given statistics

First results for scattering

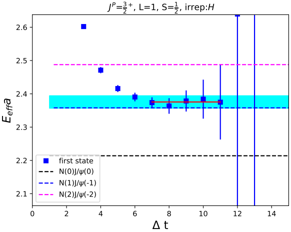
- 6 operators per channel (operators for $|p| = 1$) or 12 for channels in s wave (6 for $|p| = 0$ and 6 for $|p| = 1$) were used in simulations
- 4 (8) operators were used in analysis
- Observed system has enough energy, that lower pentaquark state could be seen
- Current results are calculated on $\approx \frac{1}{2}$ of our configurations
- Candidate channels for P_c^+ channels have J^P: $\frac{3}{2}^-$, $\frac{3}{2}^+$, $\frac{5}{2}^-$, $\frac{3}{2}^+$

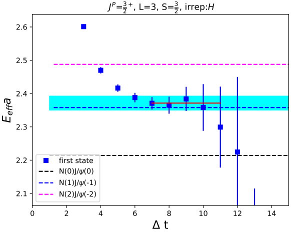
P_c^+ candidate channels: $J^P = \frac{3}{2}^-$

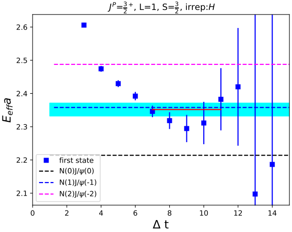




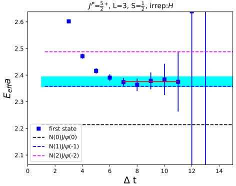
P_c^+ candidate channels: $J^P = \frac{3}{2}^+$

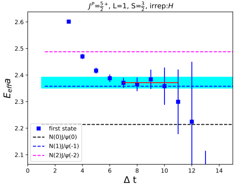


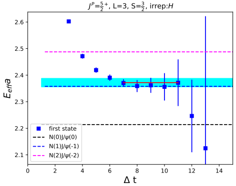


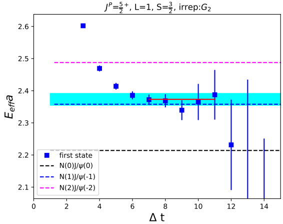


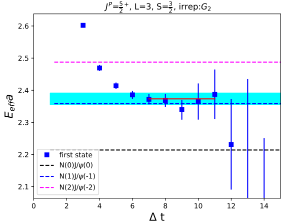
P_c^+ candidate channels: $J^P = \frac{5}{2}^+$



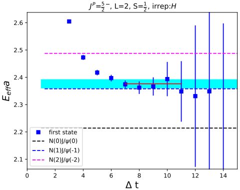


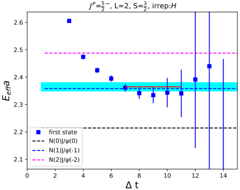


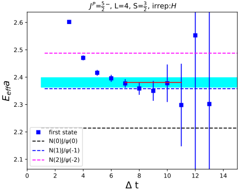


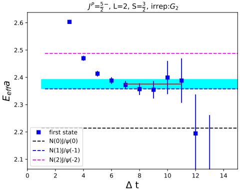


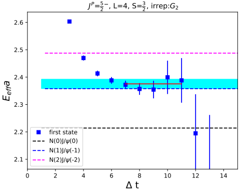
P_c^+ candidate channels: $J^P = \frac{5}{2}^-$





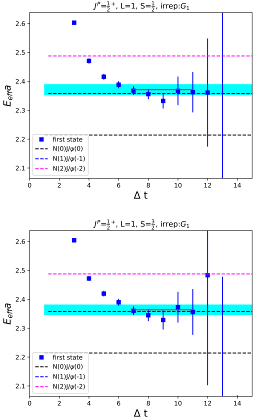




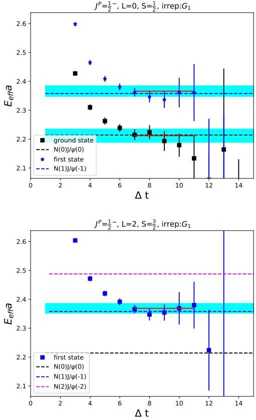


Channels which are not candidates for P_c^+

Channels with positive parity:



Channels with negative parity:



Within errors fitted energies of these channels are same as non-interacting energy of scattering of $J/\psi - N$ with momentum $|p| = 1$

Conclusion

- First preliminary results of one channel approximation for P_c^+ channels were presented.
- On given statistics and in our approximation there is no sign of energy shift, which would indicate to P_c^+ state
- P_c^+ could be a result of other neglected effects (coupled channels effect,...)
- Our future plan for this research could be:
  - Rise statistics (on all configurations of our lattice) (already in progress)
  - Add operators and results for $|p| = \sqrt{2}$ (a lot of CPU time needed)
  - Look at other channels which may form P_c^+ state $\chi_{c0} + p$ (quite easy)
  - Look at coupled channel effects (very very challenging)

References

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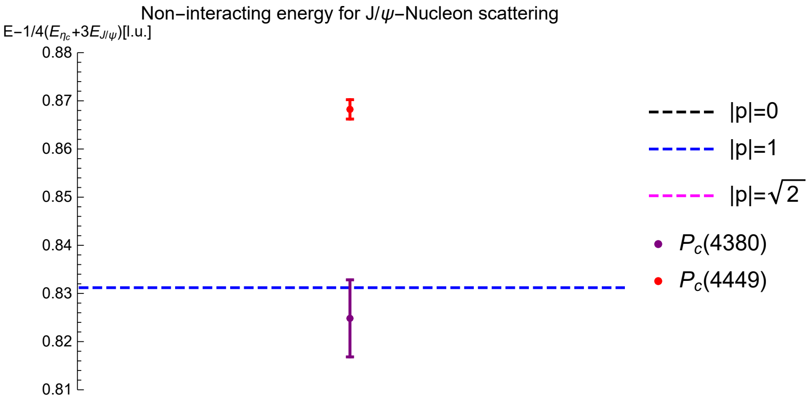
Non−interacting energy for J/ψ−Nucleon scattering
E−1/4(E_ηc+3E_J/ψ)[l.u.]
0.88
0.87
0.86
0.85
0.84
0.83
0.82
0.81
|p|=0
|p|=1
|p|=√2
Pc(4380)
Pc(4449)

J^P	L	$m_m + m_b$ [MeV]	meson	m_{meson} [MeV]	J^{PC}_{meson}	barion	m_{barion} [MeV]	J^P_{barion}
$\frac{3}{2}^-$	2^+	3921	$\eta_c(1s)$	2983.4	0^{-+}	p	938,3	$\frac{1}{2}^+$
	0^+	4034	J/ψ	3096.900	1^{--}	p	938,3	$\frac{1}{2}^+$
	0^+	4293	$D^{*0}(\bar{2}007)$	2006.85	1^-	Λ_c^+	2286.46	$\frac{1}{2}^+$
	0^+	4387	D^-	1869.59	0^-	$\Sigma_c^{++}(2520)$	2518.41	$\frac{3}{2}^+$
	1^-	4352	χ_{c0}	3414.75	0^{++}	p	938,3	$\frac{1}{2}^+$
	1^-	4448	χ_{c1}	3510.66	1^{++}	p	938,3	$\frac{1}{2}^+$
$\frac{3}{2}^+$	1^-	3921	$\eta_c(1s)$	2983.4	0^{-+}	p	938,3	$\frac{1}{2}^+$
	1^-	4034	J/ψ	3096.900	1^{--}	p	938,3	$\frac{1}{2}^+$
	1^-	4151	$\bar{D}^0$	1864.83	0^-	Λ_c^+	2286.46	$\frac{1}{2}^+$
	1^-	4293	$D^{*0}(\bar{2}007)$	2006.85	1^-	Λ_c^+	2286.46	$\frac{1}{2}^+$
	1^-	4324	D^-	1869.59	0^-	$\Sigma_c^{++}(2455)$	2453.97	$\frac{1}{2}^+$
	1^-	4387	D^-	1869.59	0^-	$\Sigma_c^{++}(2520)$	2518.41	$\frac{3}{2}^+$
	0^+	4448	χ_{c1}	3510.66	1^{++}	p	938,3	$\frac{1}{2}^+$
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	1^-	4448	χ_{c1}	3510.66	1^{++}	p	938,3	$\frac{1}{2}^+$
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	1^-	4034	J/ψ	3096.900	1^{--}	p	938,3	$\frac{1}{2}^+$
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