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Multi-Field False Vacuum Decay

POLYGONAL BOUNCE

Victor Guada

with A. Maiezza and M. Nemevšek.

Based on ArXiv: 1803.02227
PhysRevD. 99.056020

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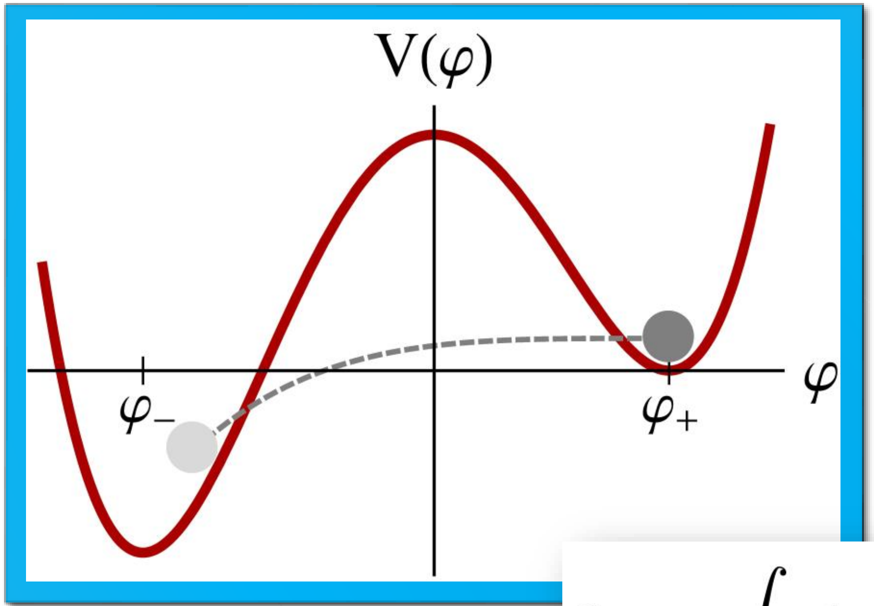
Decay Rate



[a,b]

$$\frac{\Gamma}{\mathcal{V}} = Ae^{B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = \int_{-\infty}^{\infty} d\tau L_E \equiv S_E$$

$$S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V\right]$$

[c]

$$\rho = \left(\tau^2 + |\vec{x}|^2\right)^{1/2}$$

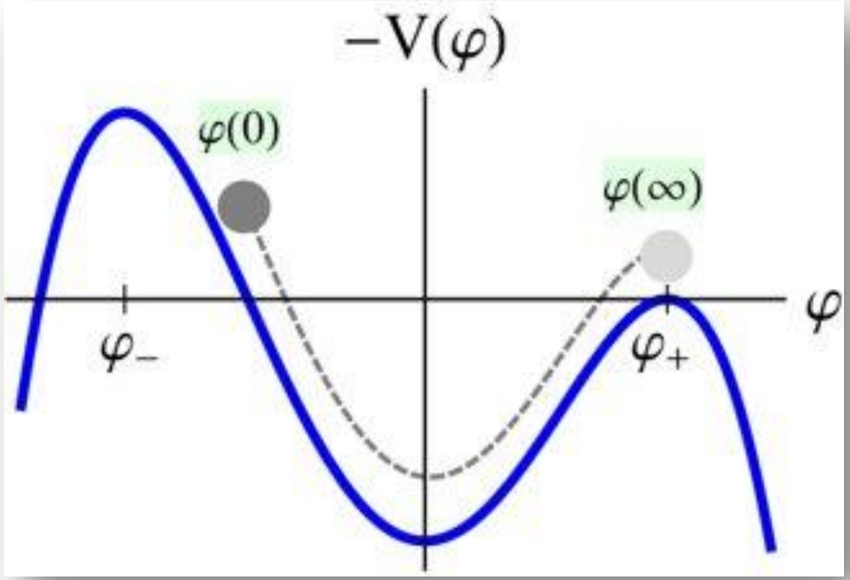
$$S_E = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^{\infty} \rho^{D-1} d\rho \left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$

a. I. Y. Kobzarev, L. B. Okun and M. B. Voloshin, Sov. J. Nucl. Phys. 20 (1975) 644
b. S. R. Coleman, Phys. Rev. D 15 (1977) 2929
c. S. R. Coleman, V. Glaser and A. Martin, Commun.Math. Phys. 58 (1978) 211

Single Field False Vacuum Decay

$$\frac{d^2\varphi}{d\rho^2} + \frac{D-1}{\rho}\frac{d\varphi}{d\rho} = V'(\varphi)$$

$$\varphi(0) = \varphi_0$$
$$\dot{\varphi}(0) = 0$$
$$\dot{\varphi}(\infty) = 0$$



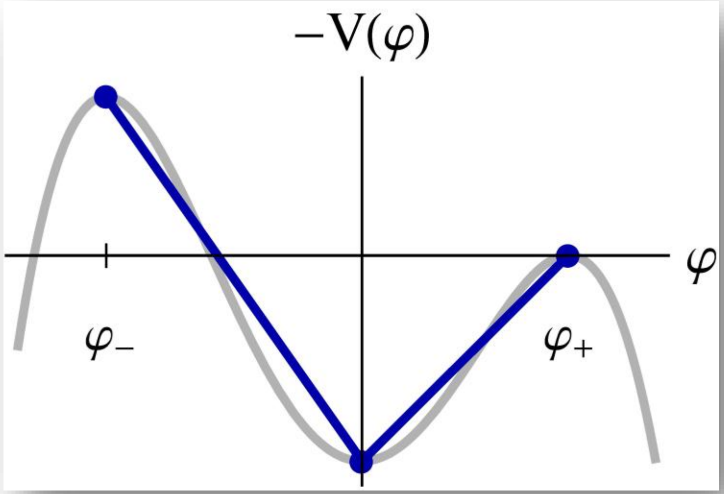
a. S. R. Coleman, V. Glaser and A. Martin, Commun.Math. Phys. 58 (1978) 211

Piecewise Linear potential[a]

$$\frac{d^2\varphi}{d\rho^2} + \frac{D-1}{\rho}\frac{d\varphi}{d\rho} = V'(\varphi)$$

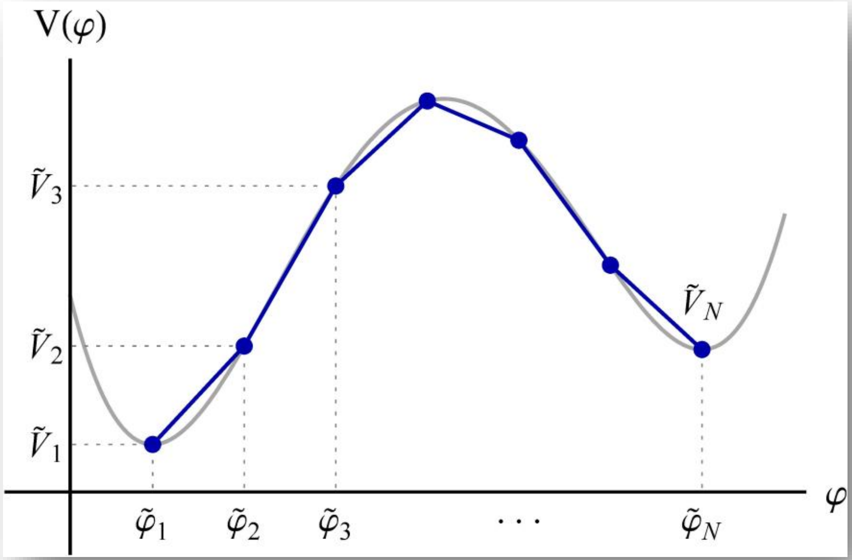
$$V(\varphi) = 8a\varphi$$

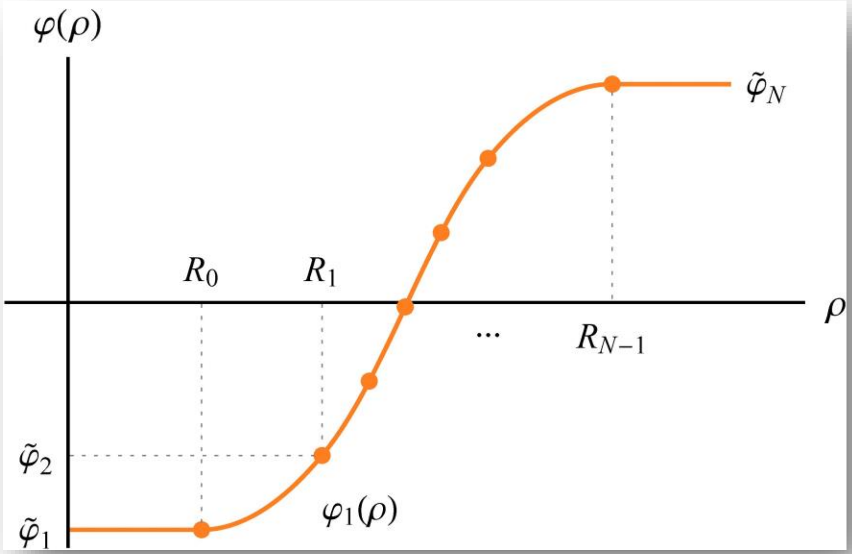
$$\varphi(\rho) = v + a\rho^2 + \frac{b}{\rho^2}$$



a. M. J. Duncan and L. G. Jensen, Phys. Lett. B 291 (1992) 109.

Polygonal Bounce



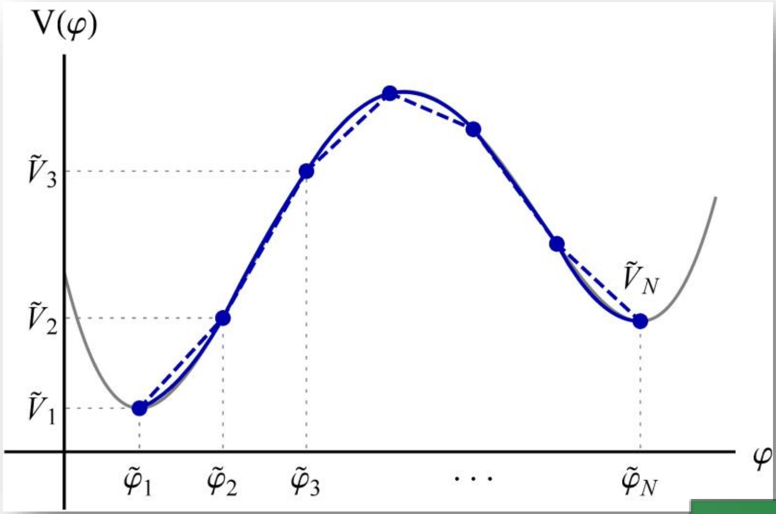


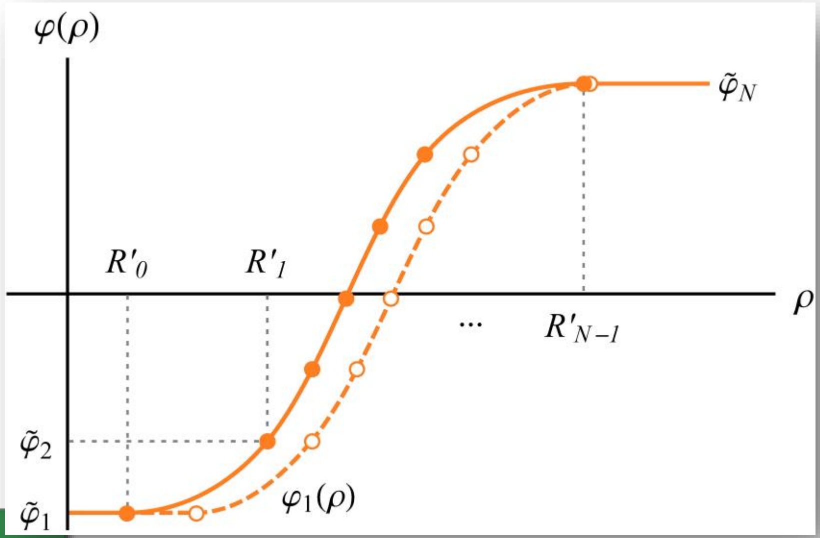
$$V_s(\varphi) = \underbrace{\left(\frac{\tilde V_{s+1} - \tilde V_s}{\tilde\varphi_{s+1} - \tilde\varphi_s}\right)}_{8\,a_s} (\varphi - \tilde\varphi_s) + \tilde V_s - \tilde V_N .$$

$$\varphi_s(\rho) = v_s + \frac{4}{D} a_s \rho^2 + \frac{2}{D-2}\frac{b_s}{\rho^{D-2}}$$

$$a_s R_s^D - \frac{D}{4}(\tilde\varphi_{s+1} - v_s) R_s^{D-2} + \frac{D}{2(D-2)} b_s = 0$$

Extending Polygonal Bounce





$$\frac{d^2\varphi}{d\rho^2} + \frac{D-1}{\rho}\frac{d\varphi}{d\rho} = V'(\varphi)$$

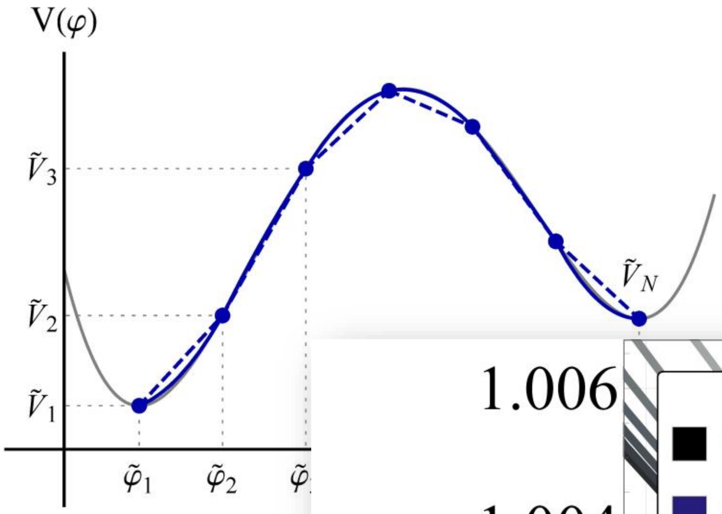
Expansions

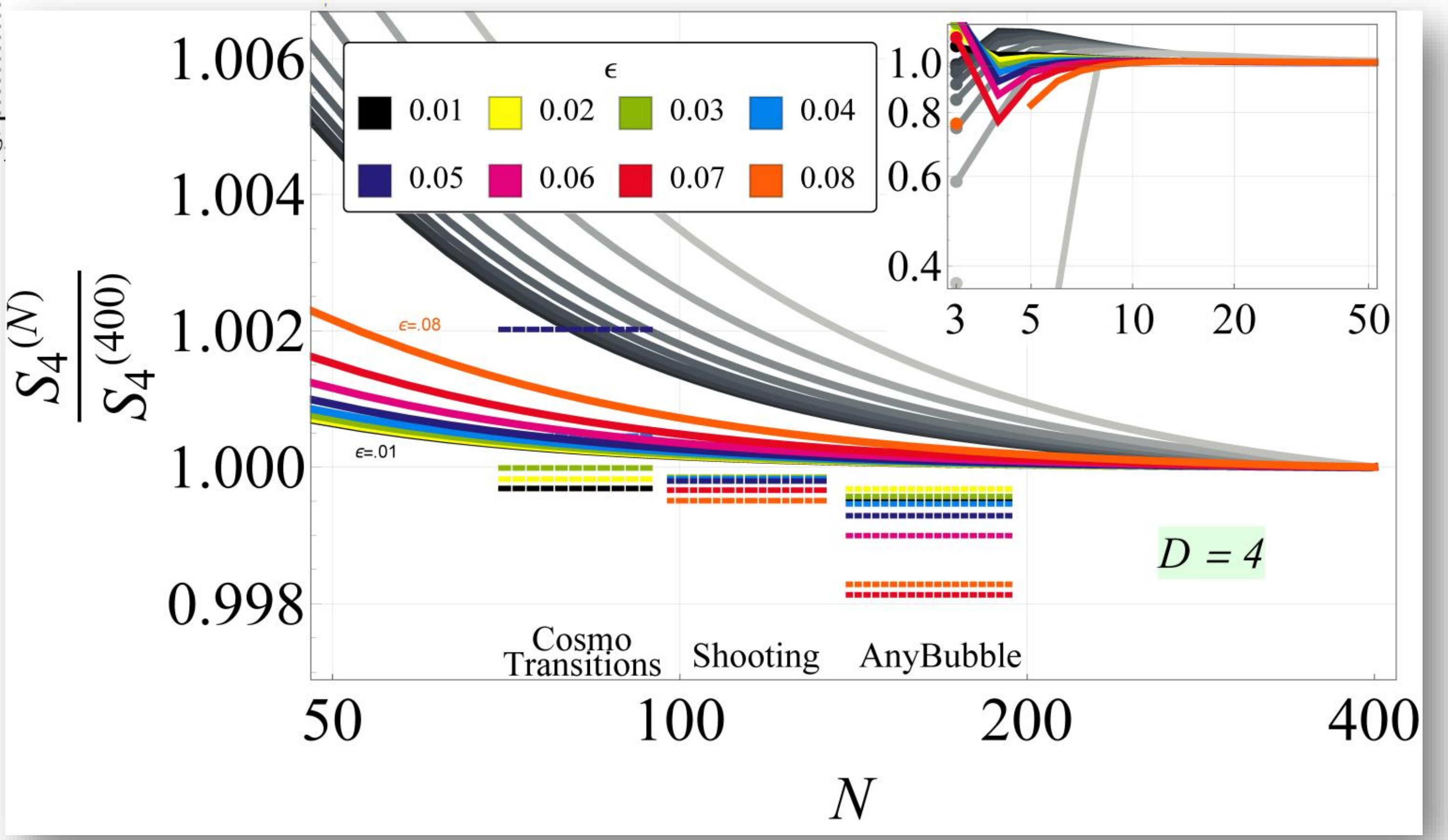
$$\varphi = \varphi_{PB} + \xi$$

$$\tilde{V}_s - \tilde{V}_N + \partial\tilde{V}_s\left(\varphi_s - \tilde{\varphi}_s\right) + \frac{\partial^2\tilde{V}_s}{2}\left(\varphi_s - \tilde{\varphi}_s\right)^2 + \dots$$

$$R_s \to R_s\left(1 + r_s\right) \qquad r_s \ll 1$$

$$\xi = \nu + \frac{2}{D-2}\frac{\beta}{\rho^{D-2}} + \frac{4}{D}\alpha\rho^2 + \mathcal{I}(\rho)$$



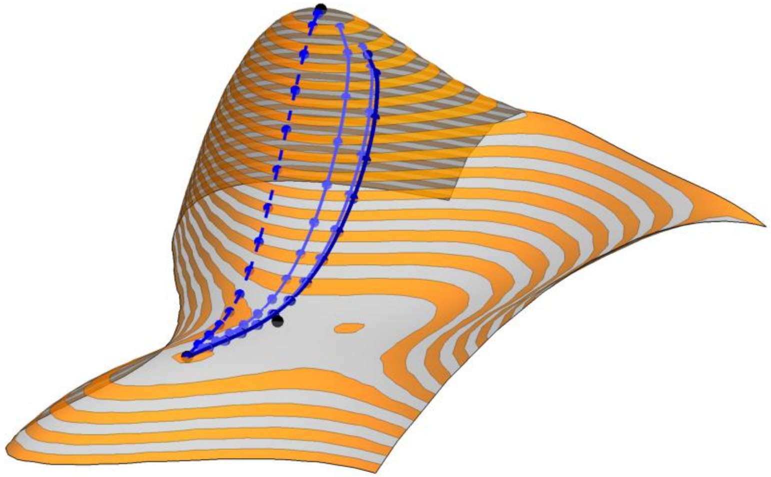


Extending Polygonal Bounce

Multi-Field Vacuum Decay

$\mathcal{O}(D)$ [a]

$$S_D = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty \rho^{D-1}\, \mathrm{d}\rho \left(\frac{1}{2} \sum_i \dot{\varphi}_i^2 + V(\varphi_i) \right)$$



$$\ddot{\varphi}_i + \frac{D-1}{\rho}\dot{\varphi}_i = d_i V$$

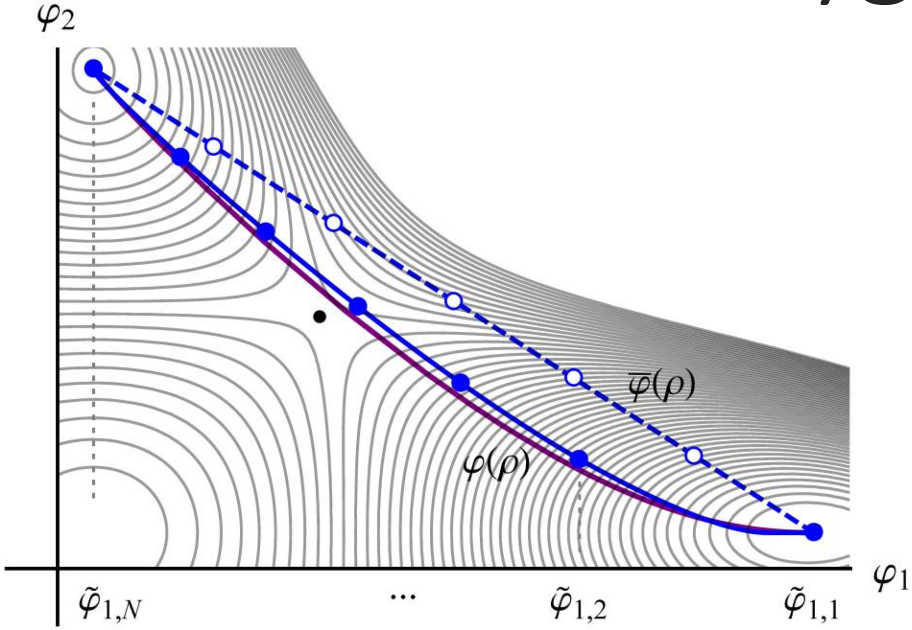
$$\varphi_i(0) = \varphi_{i0}$$

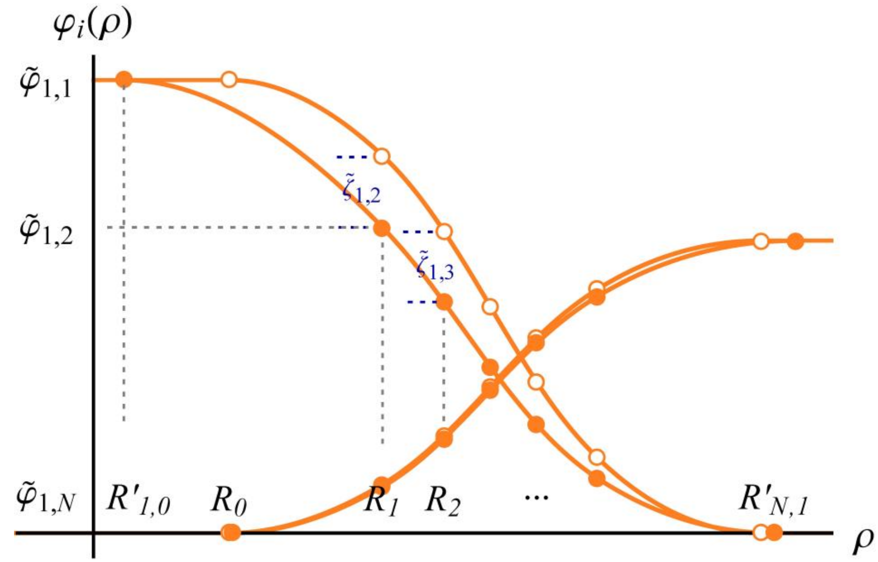
$$\dot{\varphi}_i(0) = 0$$

$$\dot{\varphi}_i(\infty) = 0$$

a. K. Blum, M. Honda, R. Sato, M. Takimoto and K. To-bioka, JHEP 1705 (2017) 109

Multi-Field Polygonal Bounce





$$\varphi_{is}(\rho) = \bar{\varphi}_{is} + \zeta_{is}$$

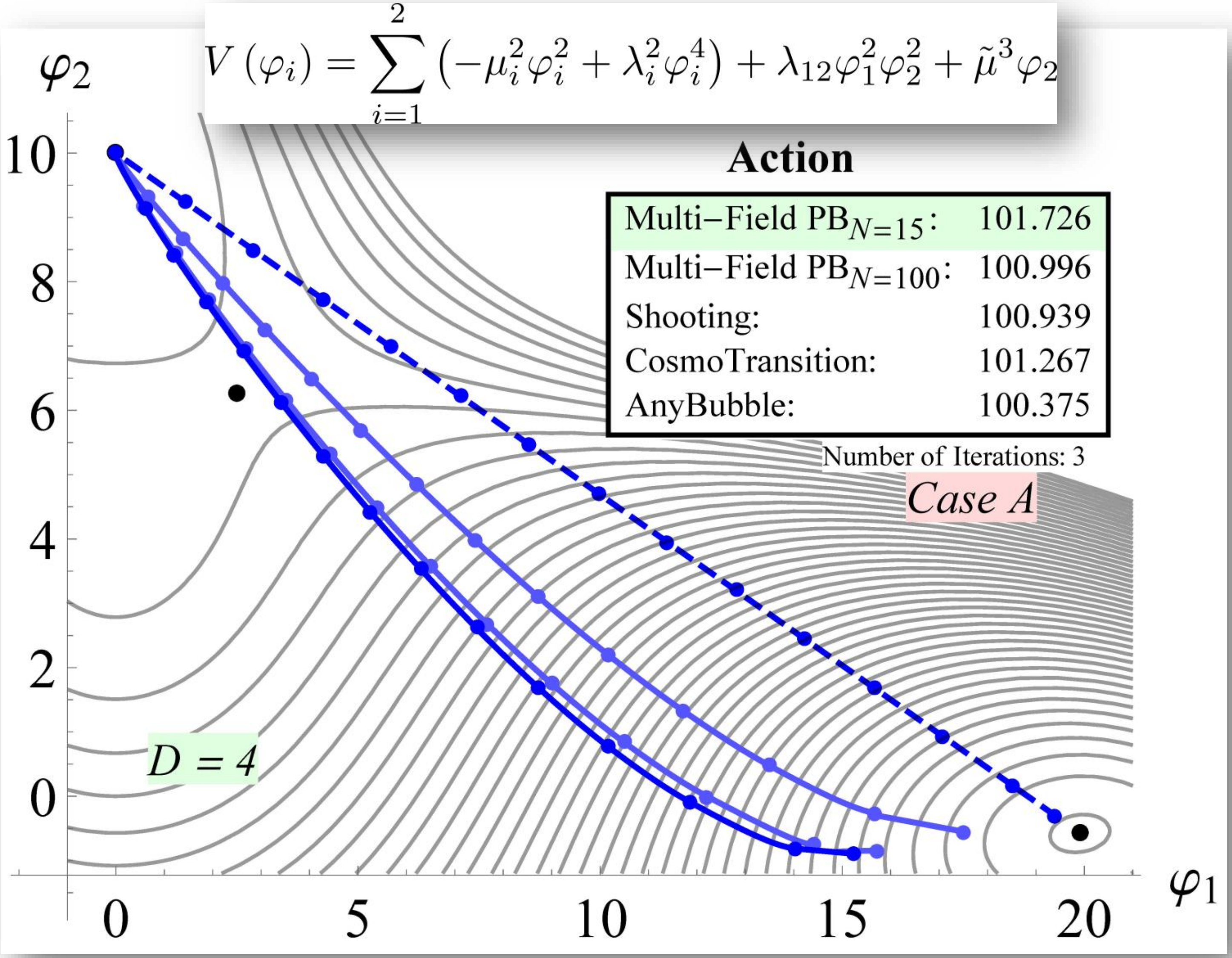
$$\underbrace{\ddot{\bar{\varphi}}_{is} + \frac{D-1}{\rho}\dot{\bar{\varphi}}_{is}}_{8\bar{a}_{is}} + \underbrace{\ddot{\zeta}_{is} + \frac{D-1}{\rho}\dot{\zeta}_{is}}_{8a_{is}} = \frac{dV}{d\varphi_i}\left(\bar{\varphi} + \zeta\right)$$

$$\underbrace{\ddot{\bar{\varphi}}_{is} + \frac{D-1}{\rho}\dot{\bar{\varphi}}_{is}}_{8\bar{a}_{is}} + \underbrace{\ddot{\zeta}_{is} + \frac{D-1}{\rho}\dot{\zeta}_{is}}_{8a_{is}} = \frac{dV}{d\varphi_i}\left(\bar{\varphi} + \zeta\right)$$

$$\frac{dV}{d\varphi_i} \simeq \frac{1}{2}\left(d_i\tilde{V}_s + d_i\tilde{V}_{s+1} + d^2_{ij}\tilde{V}_s\tilde{\zeta}_{js} + d^2_{ij}\tilde{V}_{s+1}\tilde{\zeta}_{js+1}\right)$$

$$\zeta_{is} = v_{is} + \frac{2}{D-2}\frac{b_{is}}{\rho^{D-2}} + \frac{4}{D}a_{is}\rho^2$$

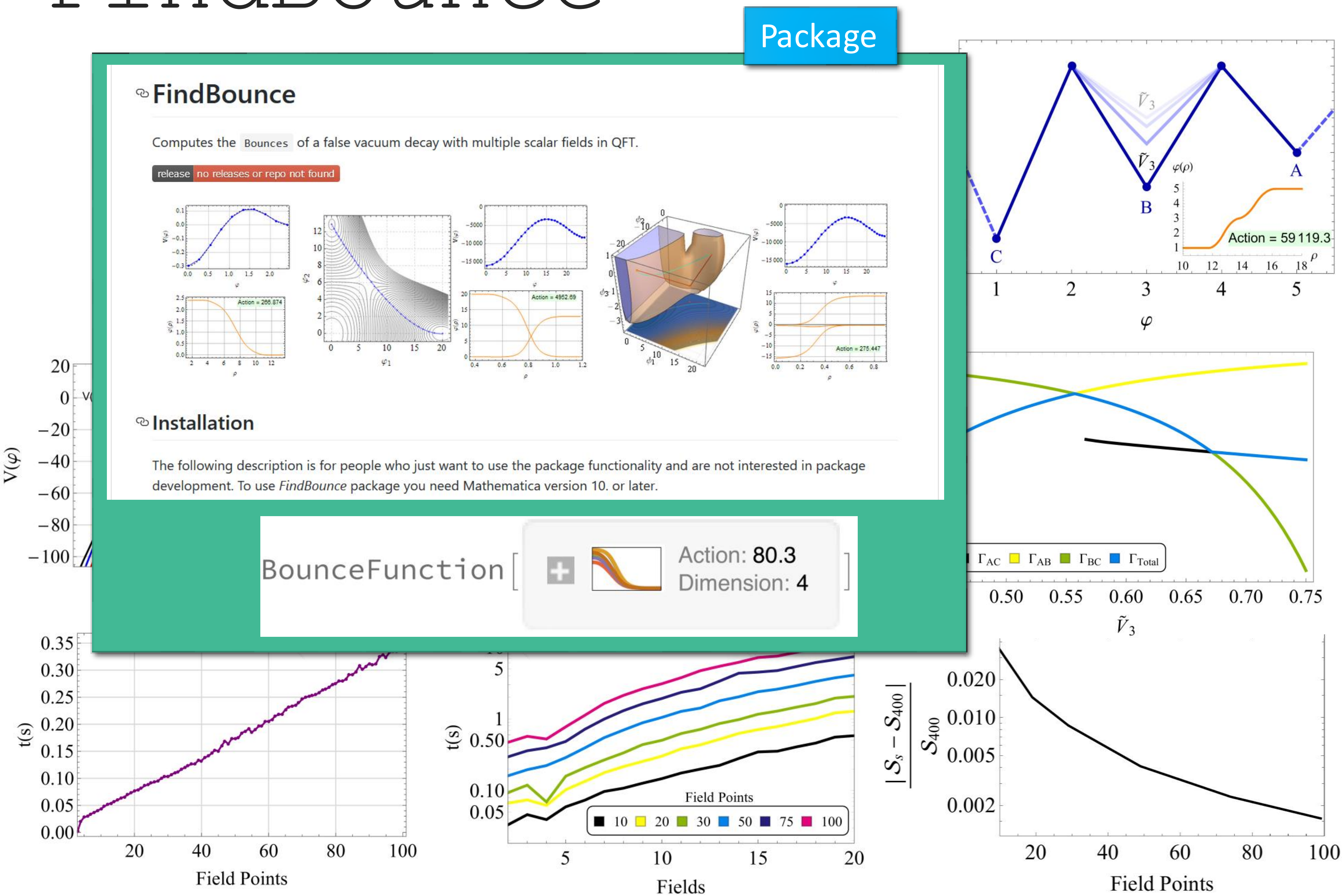
a. S. Akula, C. Balázs and G. A. White, Eur. Phys. J. C 76(2016) no.12, 681



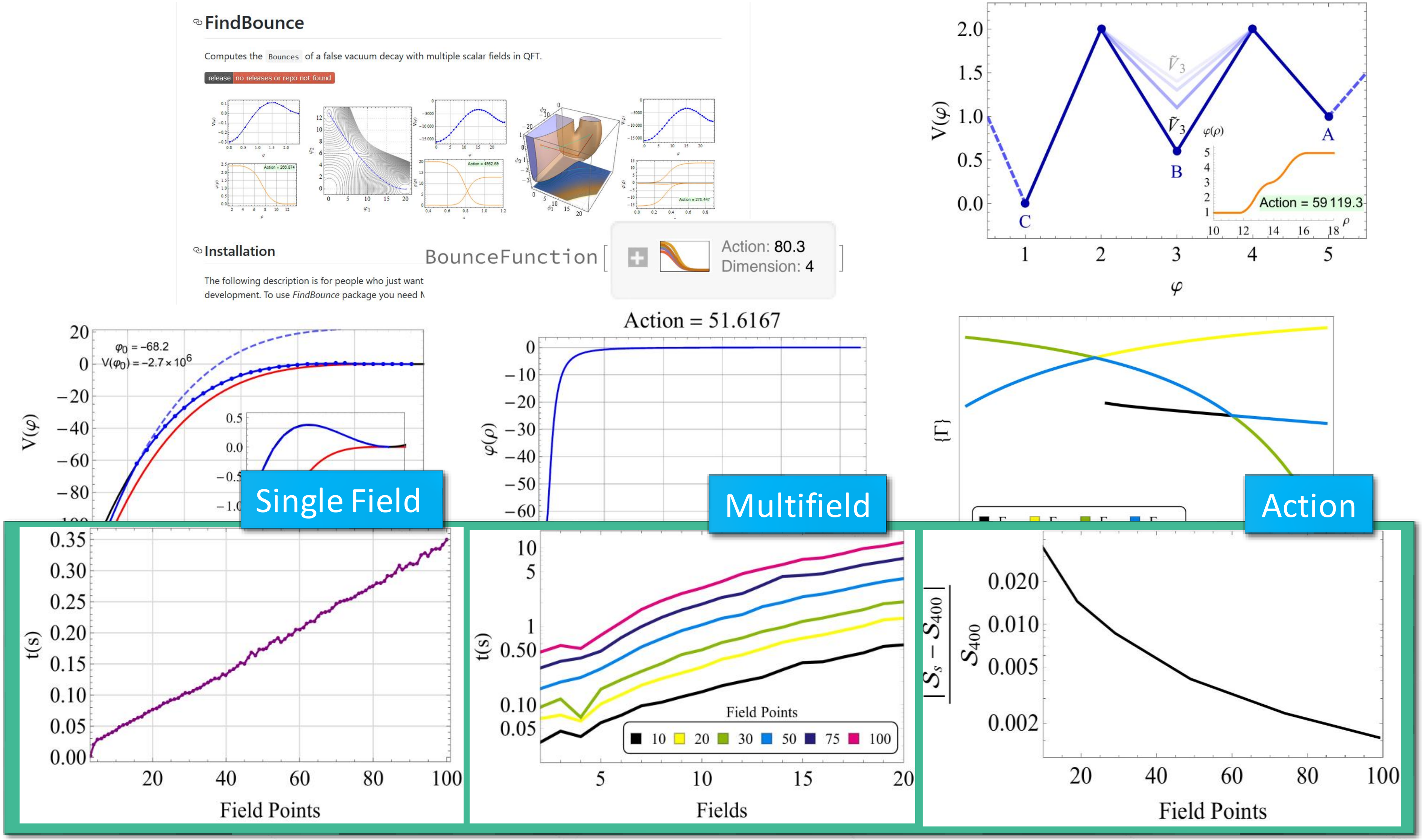
a. J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D60 (1999) 105035.
C. L. Wainwright, Comput. Phys. Commun. 183 (2012) 2006.
b. A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701(2017) no.01, 051
c. P. Athron, C. Balázs, M. Bardsley, A. Fowlie, D. Harries and G. White.

Multi-Field False Vacuum Decay

FindBounce

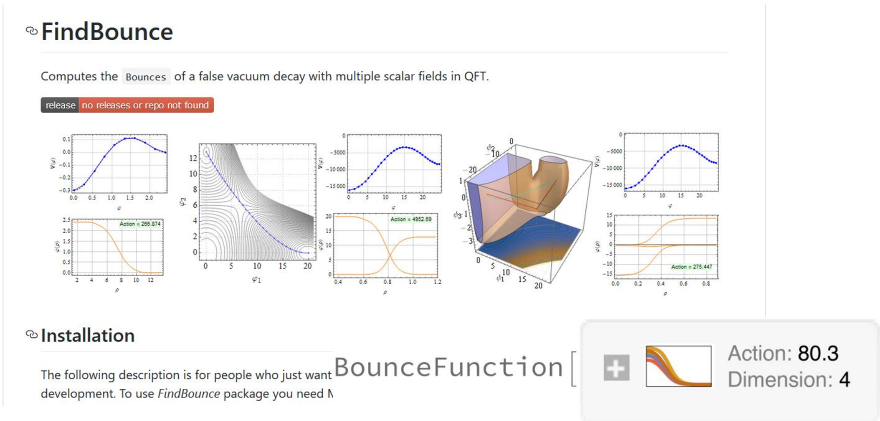

Package
FindBounce
Computes the Bounces of a false vacuum decay with multiple scalar fields in QFT.
release no releases or repo not found
Installation
The following description is for people who just want to use the package functionality and are not interested in package development. To use FindBounce package you need Mathematica version 10. or later.
BounceFunction[Action: 80.3 Dimension: 4]
Action = 59 119.3
C
B
A
φ
φ(ρ)
ρ
V(φ)
t(s)
Field Points
Fields
Γ_AC Γ_AB Γ_BC Γ_Total
Field Points
10 20 30 50 75 100

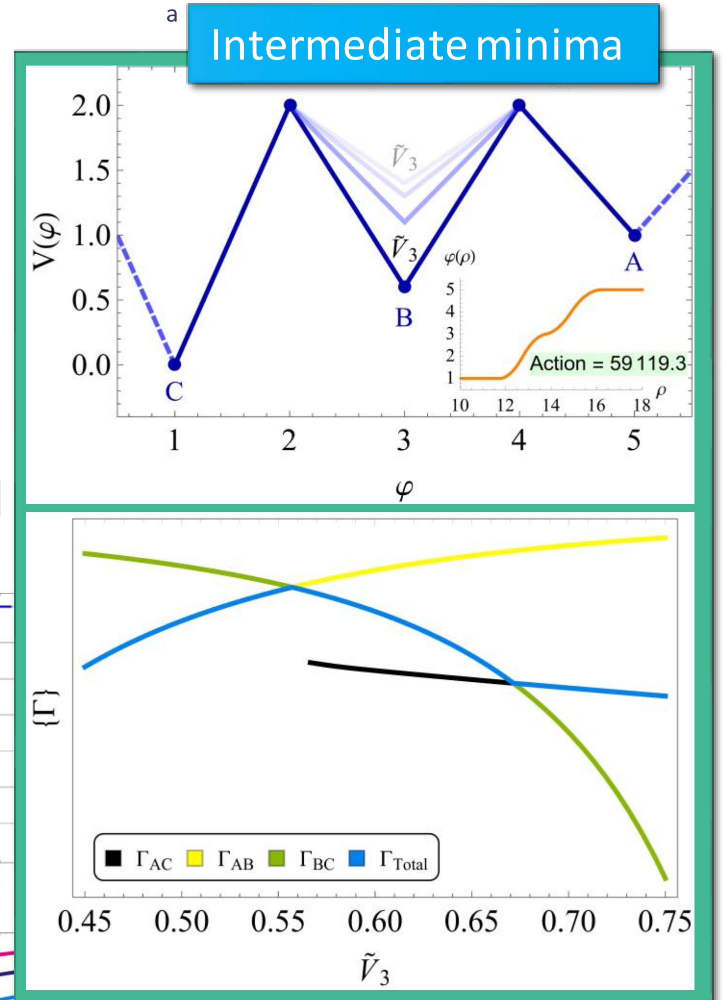
FindBounce

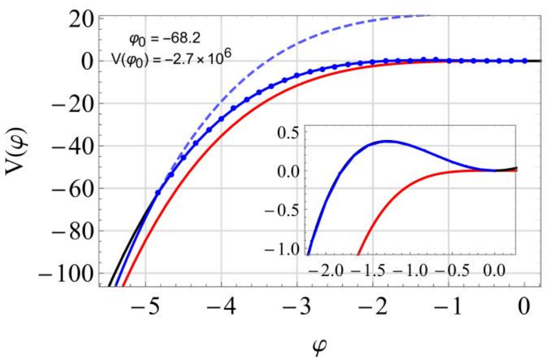


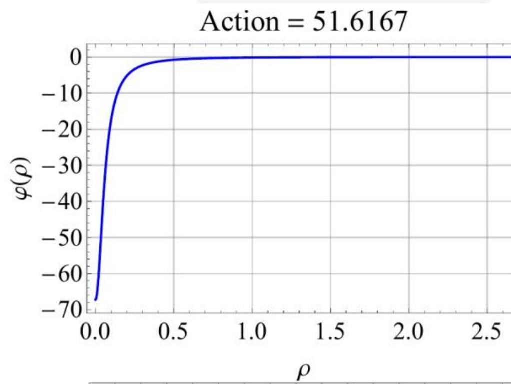
FindBounce
Computes the Bounces of a false vacuum decay with multiple scalar fields in QFT.
release no releases or repo not found
Installation
The following description is for people who just want
development. To use FindBounce package you need
BounceFunction[Action: 80.3 Dimension: 4]
Action = 59 119.3
Action = 51.6167
Single Field
Multifield
Action
t(s)
Field Points
Fields
10 20 30 50 75 100

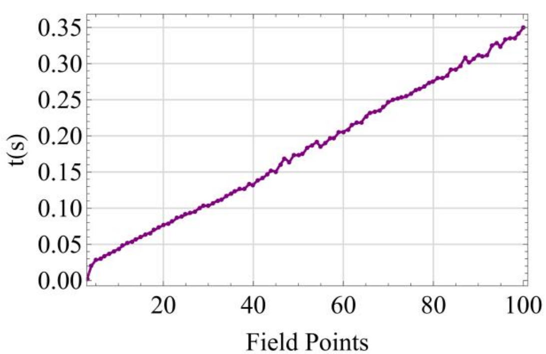
FindBounce

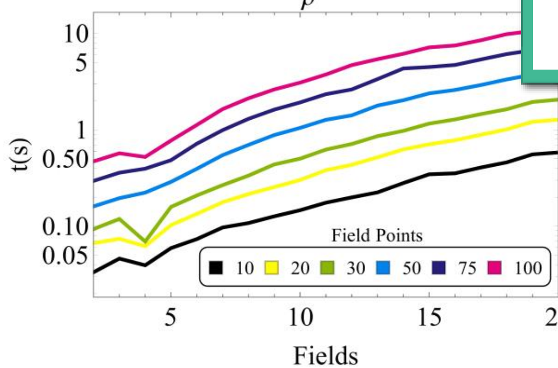


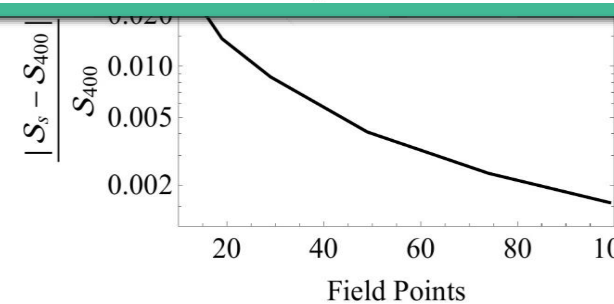






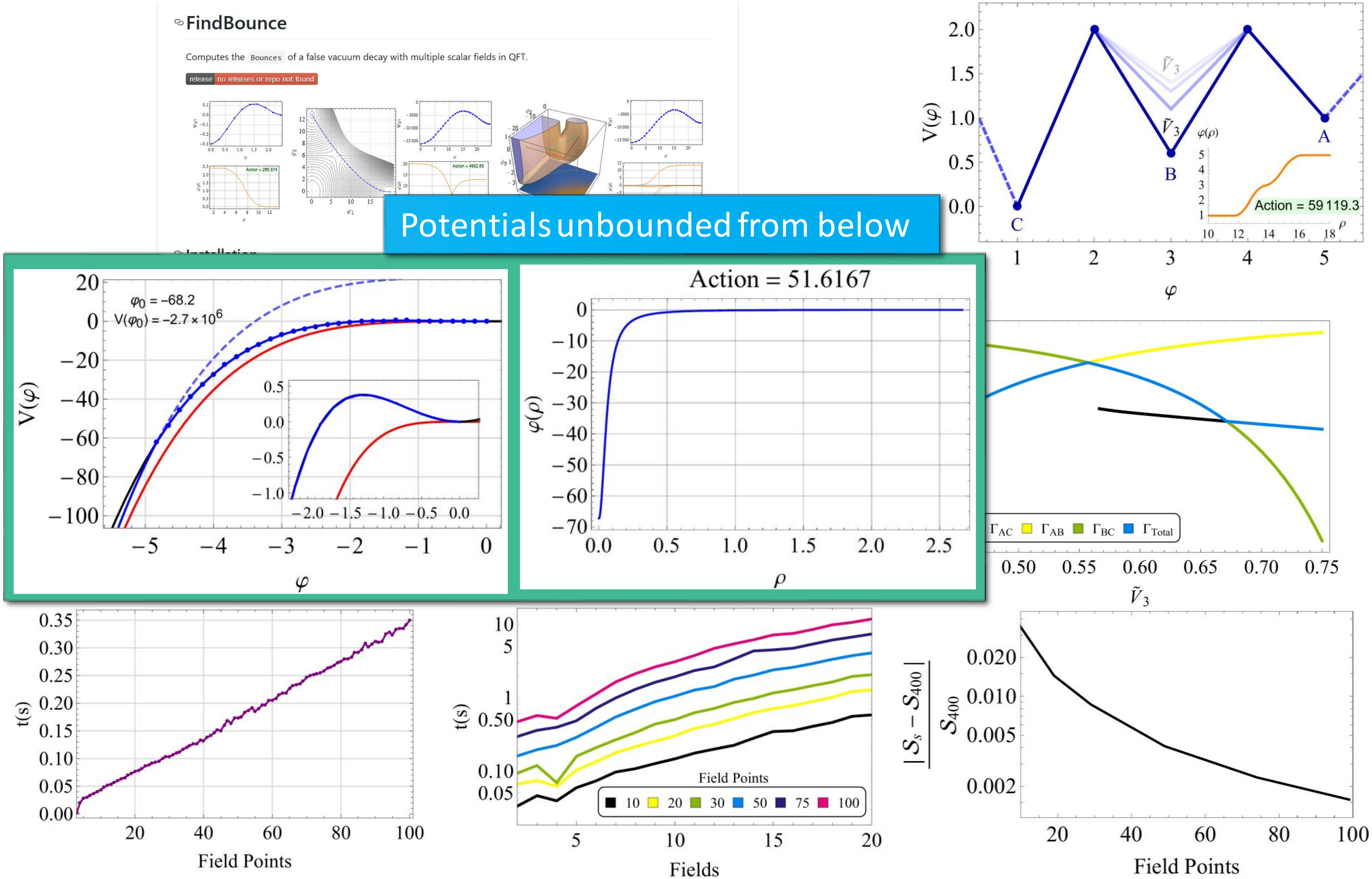






a. A. R. Brown and A. Dahlen, Phys. Rev. D 84 (2011) 105004

FindBounce


FindBounce
Computes the Bounces of a false vacuum decay with multiple scalar fields in QFT.
release no releases or repo not found
Potentials unbounded from below
$\varphi_0 = -68.2$
$V(\varphi_0) = -2.7 \times 10^6$
V(φ)
φ
Action = 51.6167
φ(ρ)
ρ
C
B
A
Action = 59 119.3
$\tilde{V}_3$
Γ_{AC} Γ_{AB} Γ_{BC} Γ_{Total}
t(s)
Field Points
Fields
10 20 30 50 75 100
$\frac{|S_s - S_{400}|}{S_{400}}$

Conclusions

*We developed a **semi-analytical**, **simple** and **fast** approach to compute the false vacuum **decay rate** for arbitrary potentials with any number of fields and any space-time dimensions **up to the desired precision**.*

*Provides an **analytical insight** in describing the **vacuum structure** of the potential, thermodinamical **bubble nucleations** and it's related spectrum of **gravitational waves** at the early Universe.*

Back up Slides

Pre-factor A on Polygonal Bounce

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-S_E}\left[1 + \mathcal{O}(\hbar)\right]$$

[a]

$$\Gamma = \left(\frac{S_4}{2\pi}\right)^2 \left|\frac{\det'(-\partial^2 + V''(\varphi(\rho)))}{\det(-\partial^2 + V''(\varphi_+))}\right|^{-1/2} e^{-S_4-\delta_4}$$

$\mathcal{O}(4)$ →

$$\mathcal{O}_l = -\frac{d^2}{d\rho^2} - \frac{3}{\rho}\frac{d}{d\rho} + \frac{l(l+1)}{\rho^2} + V''(\rho) + 1$$

$$V''(\rho) = -3\varphi(\rho) + \frac{3\alpha}{2}\varphi^2(\rho)$$

[b,c]

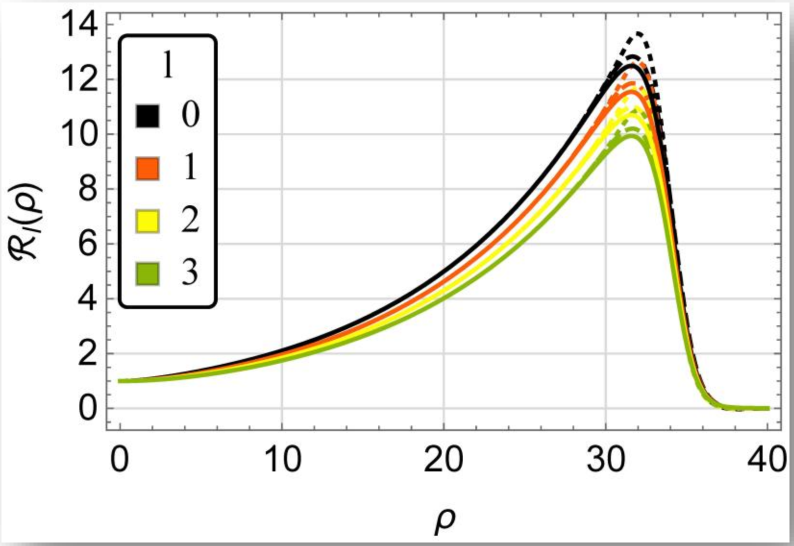
$$\frac{\det \mathcal{O}_l}{\det \mathcal{O}_l^{\text{free}}} = \mathcal{R}_l(\rho = \infty)^{(l+1)^2} \qquad \mathcal{R}_l(\rho) = \frac{\psi_l(\rho)}{\psi_l^{\text{free}}(\rho)}$$

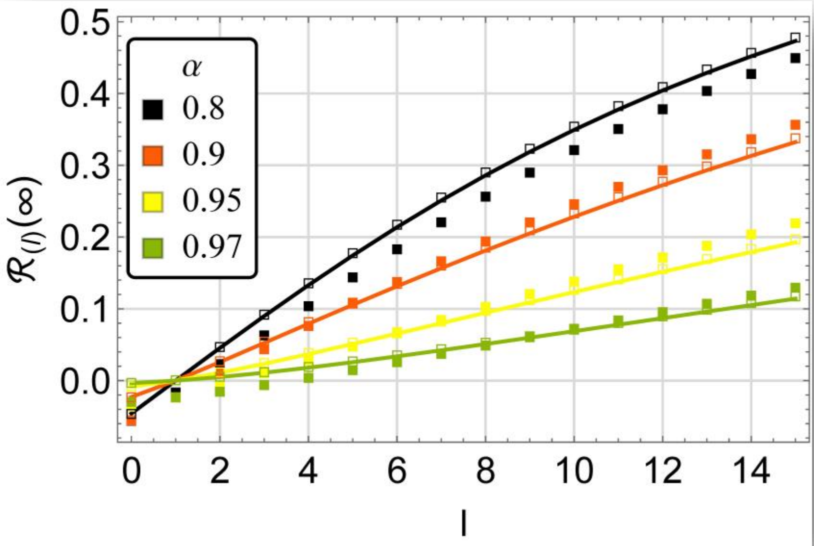
a. C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D 16 (1977) 1762
b. G. V. Dunne and H. Min, Phys. Rev. D 72 (2005) 125004.
c. I. M. Gelfand and A. M. Yaglom, J. Math. Phys. 1 (1960) 48.

Pre-factor A on Polygonal Bounce

$$\mathcal{R}_l(\rho) = \frac{\psi_l(\rho)}{\psi_l^{\text{free}}(\rho)}$$

$$\frac{\det \mathcal{O}_l}{\det \mathcal{O}_l^{\text{free}}} = \mathcal{R}_l(\rho = \infty)^{(l+1)^2}$$





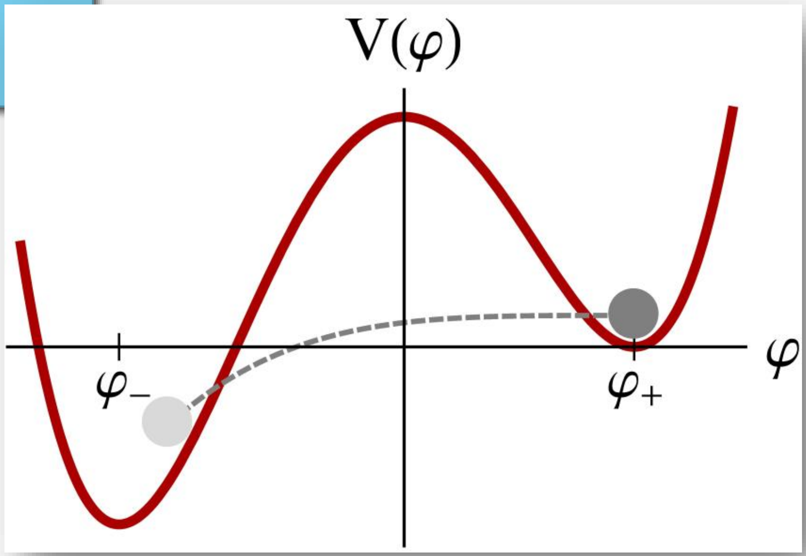
$$\frac{\Gamma}{\mathcal{V}} = Ae^{-S_E}\left[1 + \mathcal{O}(\hbar)\right]$$

$\alpha \to 1$ Thin wall limit

Around 10% contribution

Introduction

- Different context of physics
- Forbidden at the classical level
- First-order phase transition

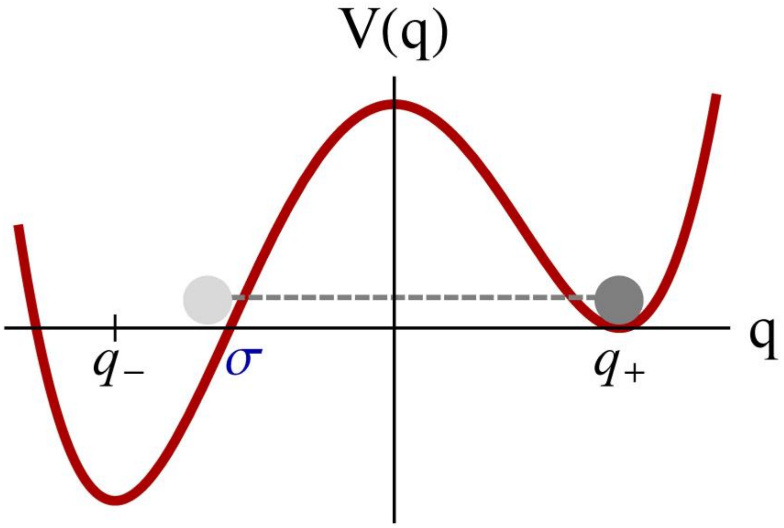


a. I. Y. Kobzarev, L. B. Okun and M. B. Voloshin, Sov. J. Nucl. Phys. 20 (1975) 644
b. S. R. Coleman, Phys. Rev. D 15 (1977) 2929
c. S. R. Coleman, V. Glaser and A. Martin, Commun. Math. Phys.
d. A. D. Linde, Phys. Lett. 100B (1981) 37. Nucl. Phys. B 216 (1983) 421

Tunneling in Quantum Mechanics

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$

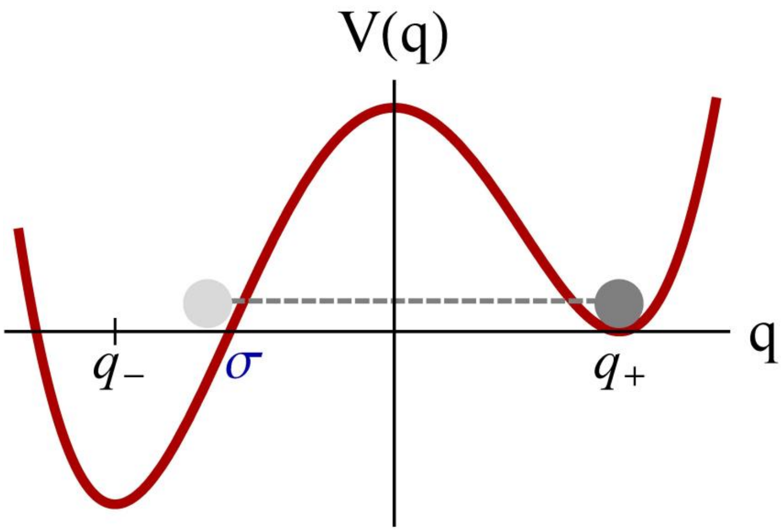


$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

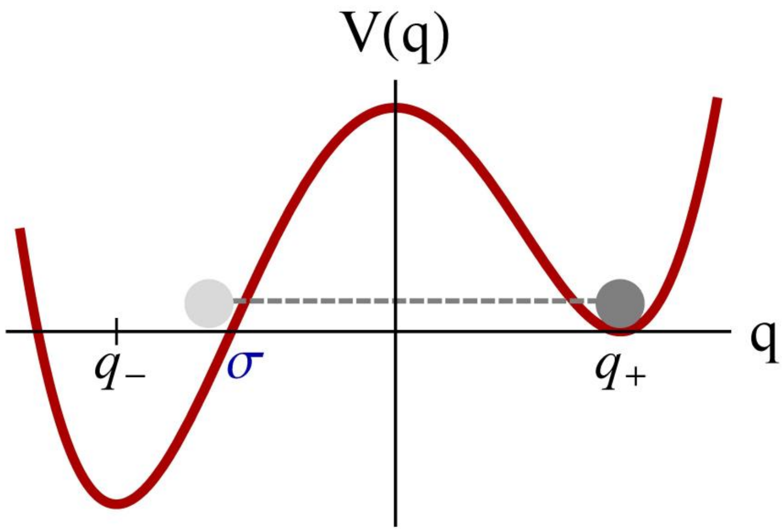
$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$

Minimum

$$\delta \int_{q_+}^{\sigma} dq\,(2V)^{1/2} = 0$$



$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1+\mathcal{O}\left(\hbar\right)\right]$$



$$B = 2\int_{q_+}^{\sigma} dq\,(2V)^{1/2}$$

Minimum

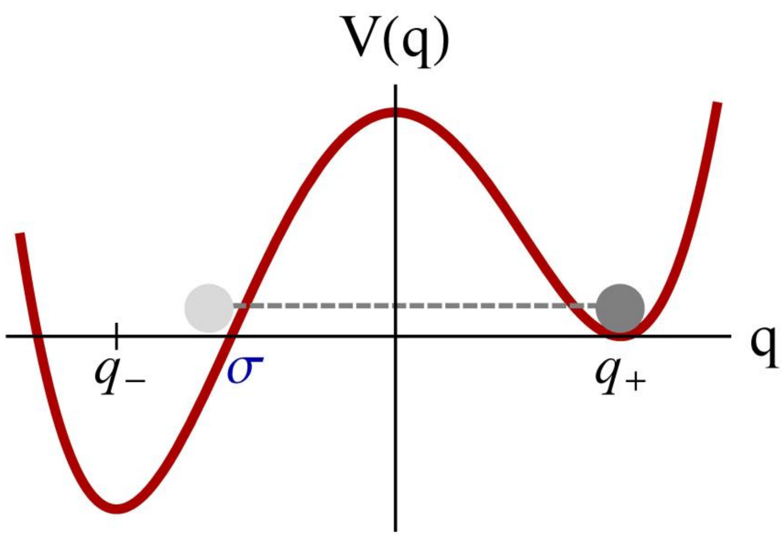
$$\delta\int_{q_+}^{\sigma} dq\,(2V)^{1/2} = 0$$

$$\delta\int_{q_+}^{\sigma} dq\,\left[2\left(E-V\right)\right]^{1/2} = 0$$

$$\frac{1}{2}\frac{dq}{dt}\cdot\frac{dq}{dt} + V = E$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,\left(2V\right)^{1/2}$$



$$\delta\int_{q_+}^{\sigma} dq\,\left(2V\right)^{1/2} = 0$$

$$\delta\int_{q_+}^{\sigma} dq\,\left[2\left(E - V\right)\right]^{1/2} = 0$$

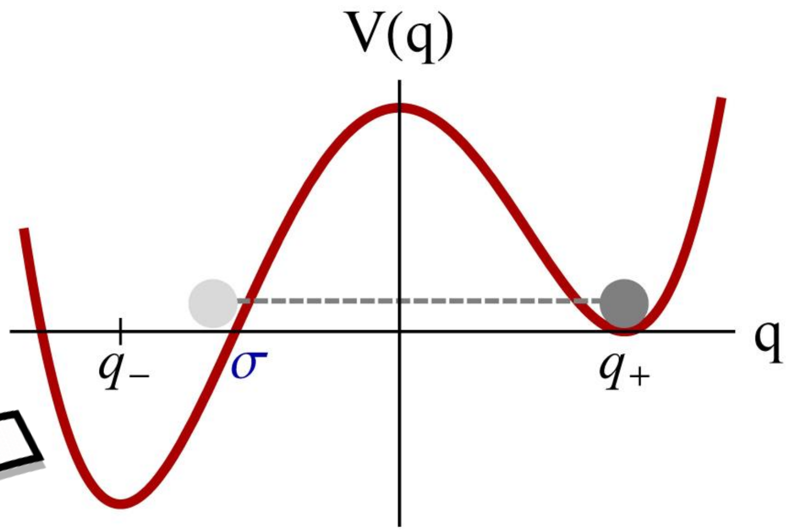
$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

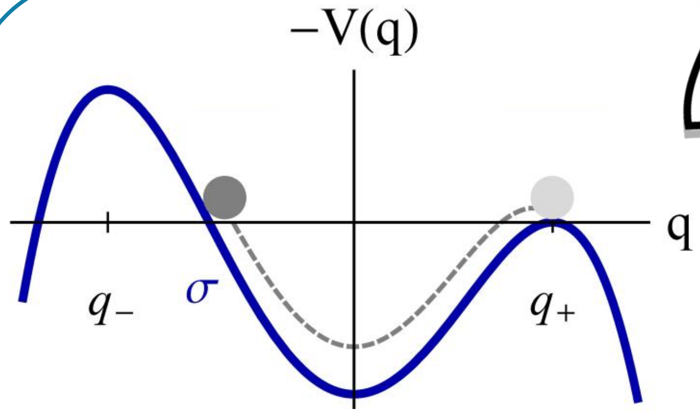
$$t \to i\tau$$

$$\frac{1}{2}\frac{dq}{dt}\cdot\frac{dq}{dt} + V = E$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\left(2V\right)^{1/2}$$





$$t \to i\tau$$

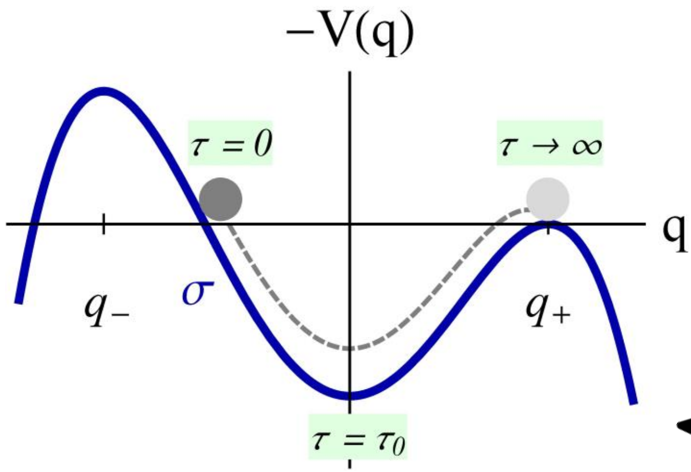
$$\delta\int_{q_+}^{\sigma} dq\left[2\left(E - V\right)\right]^{1/2} = 0$$

$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

$$\frac{1}{2}\frac{dq}{dt}\cdot\frac{dq}{dt} + V = E$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,\left(2V\right)^{1/2}$$



$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

$$\left.\frac{dq}{d\tau}\right|_{\tau_\sigma=0} = 0$$

$$\lim_{\tau\to\pm\infty} q = q_+$$

$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

$$B = 2\int_{q_+}^{\sigma} dq\,\left(2V\right)^{1/2}$$

$$\left.\frac{dq}{d\tau}\right|_{\tau_\sigma=0} = 0$$

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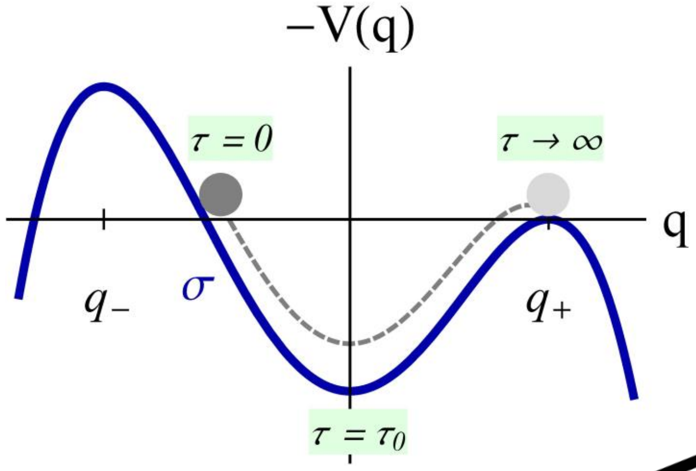


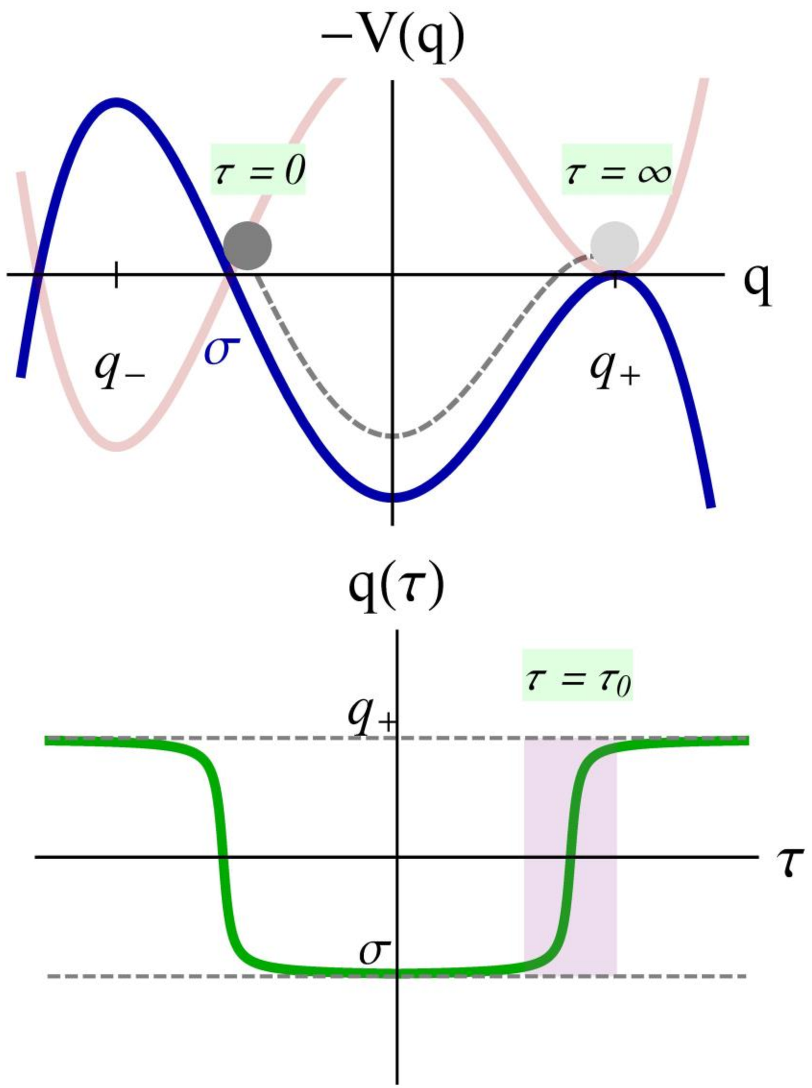
$$\int_{q_+}^{\sigma} dq\,\left(2V\right)^{1/2} = \int_{-\infty}^{0} d\tau L_E$$

$$\frac{1}{2}\frac{dq}{d\tau}\cdot\frac{dq}{d\tau} - V = 0$$

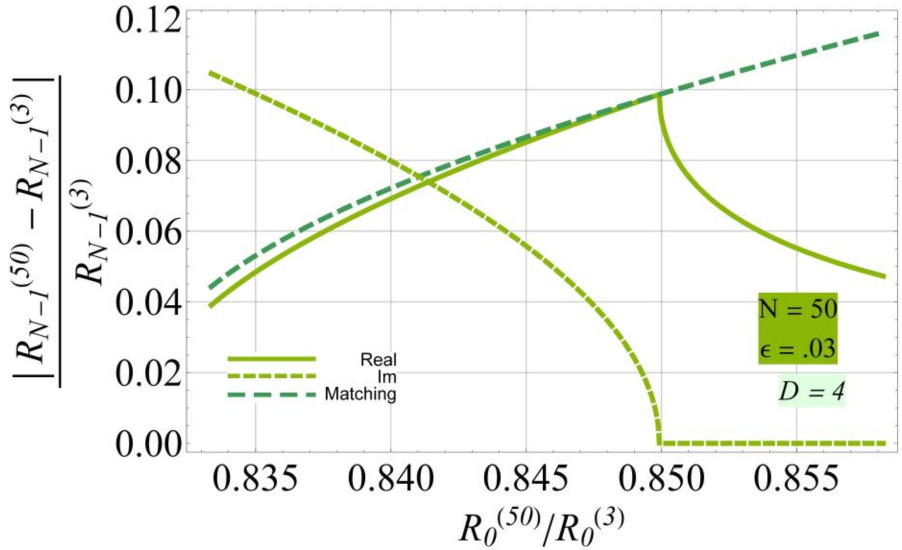
False Vacuum Decay

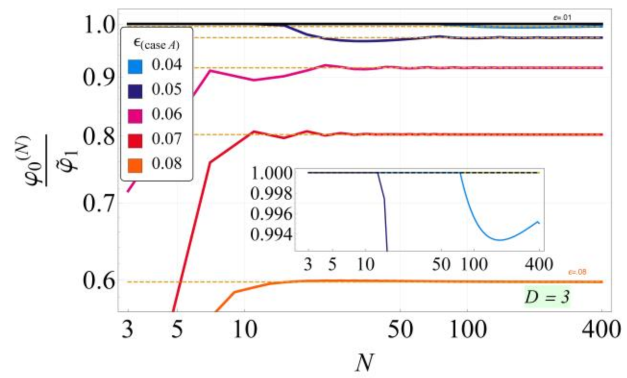
$$\frac{\Gamma}{\mathcal{V}} = Ae^{-B}\left[1 + \mathcal{O}\left(\hbar\right)\right]$$

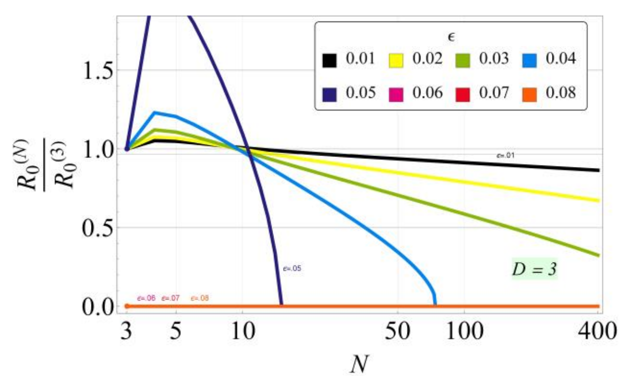
$$B = \int_{-\infty}^{\infty} d\tau L_E \equiv S_E$$



Parameters: Polygonal Bounce







Some Bounce Properties

Rescaling

$$V \to g^a V$$

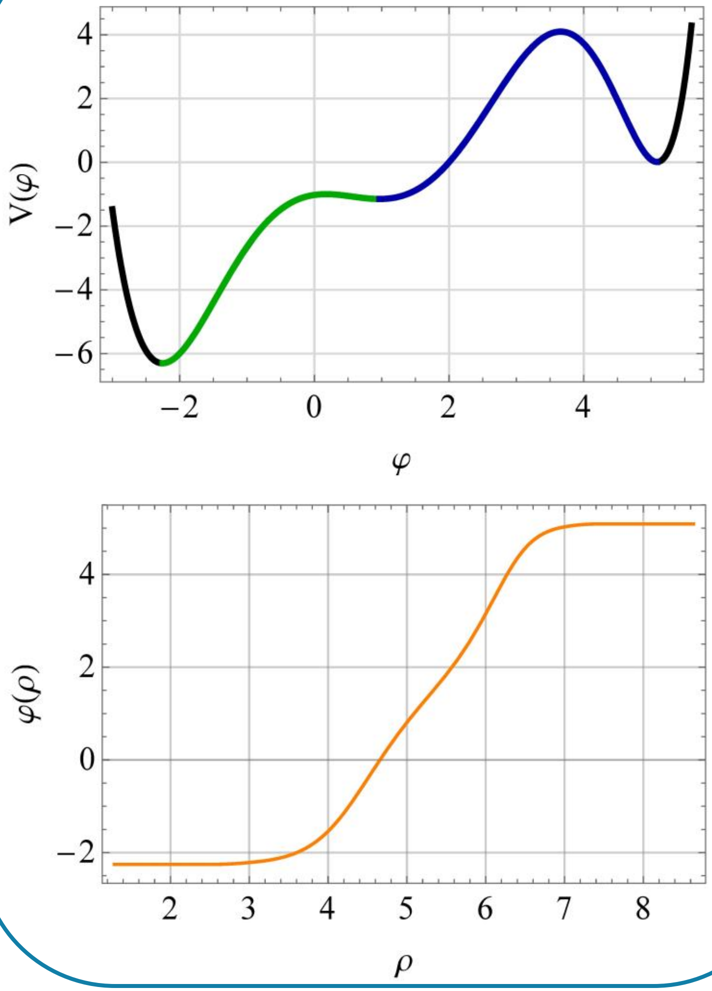
$$\phi \to g^b \phi$$

$$\rho \to g^c \rho$$

$$S_E \to g^{4b-a} S_E$$

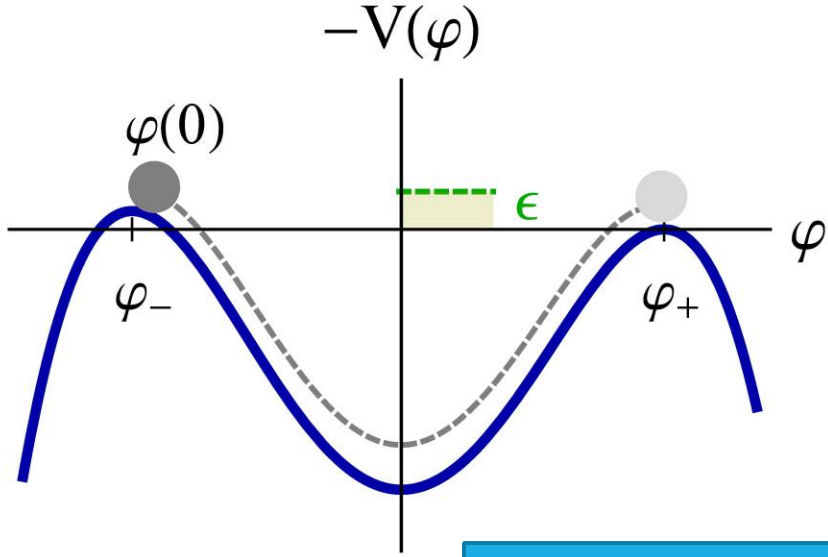
$$2c = 2b - a$$

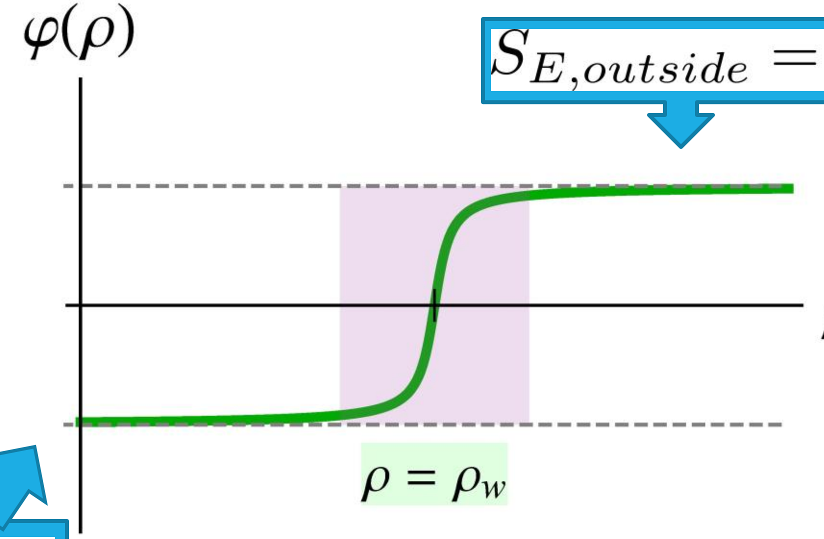
Multiple-Solutions



Thin wall approximation

$$D = 4 \Rightarrow \frac{d^2\varphi}{d\rho^2} + \frac{3}{\rho}\frac{d\varphi}{d\rho} = V'(\varphi)$$





$$S_{E,outside} = 0$$

$$S_{E,inside} = -\frac{\pi^2}{2}\rho_w^4\epsilon$$

$$S_{E,wall} = 2\pi^2\rho_w^3 S_1$$

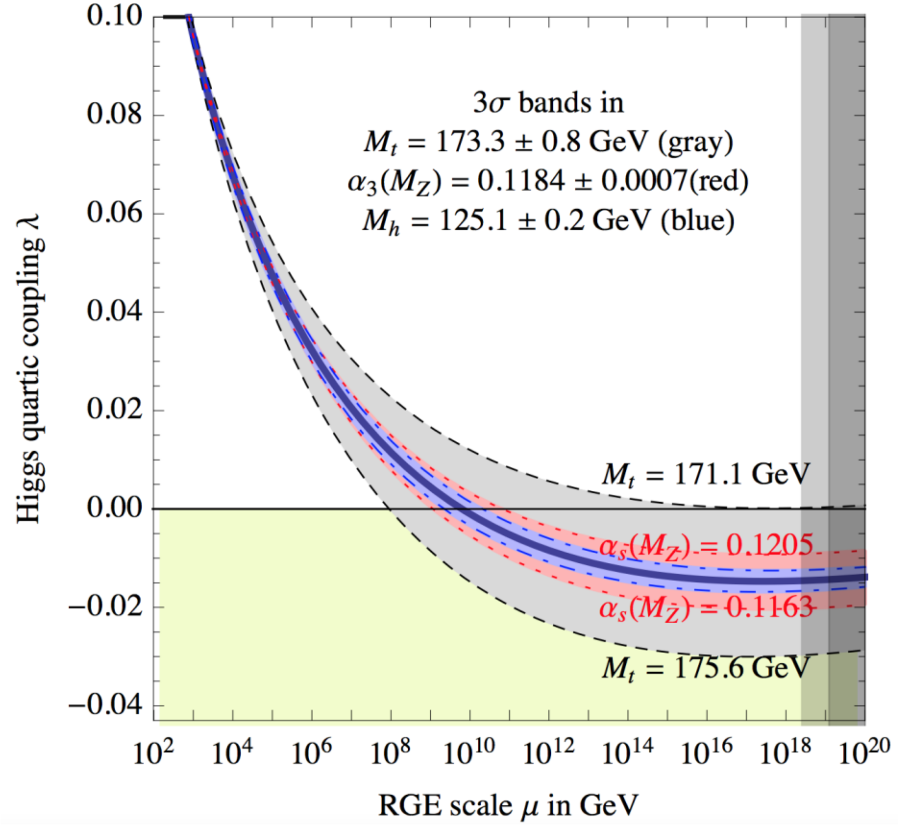
$$\frac{dS_E}{dR} = 0 \Rightarrow R = \frac{3S_1}{\epsilon}$$

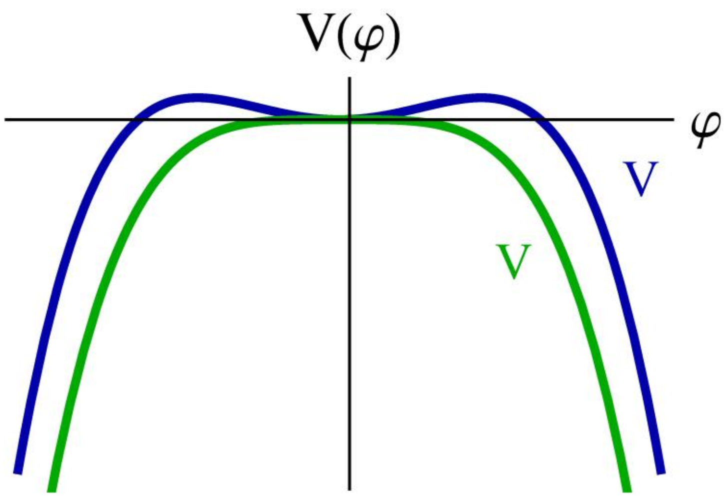
a. S. R. Coleman, Phys. Rev. D 15 (1977) 2929

Higgs Potential

$$V = -\mu^2|\Phi^2| + \lambda|\Phi^4| = \frac{1}{2}m^2h^2 + \sqrt{m\lambda}\,h^3 + \frac{1}{4}\lambda h^4 + \ldots$$

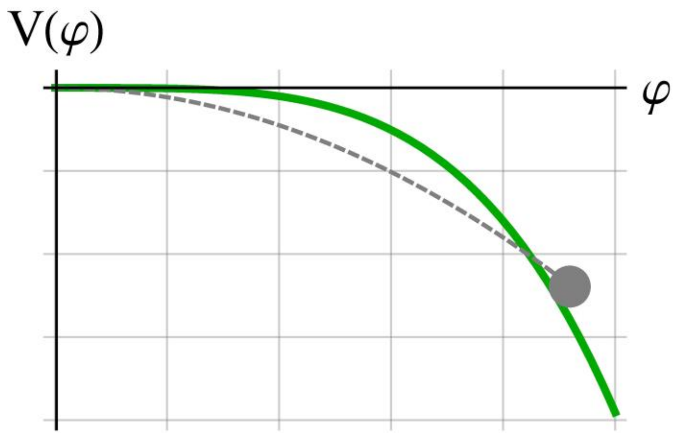
$$V\left(h \gg v\right) \cong \frac{1}{4}\lambda_{eff}\left(h\right)h^4$$





a. G. Isidori, V. S. Rychkov, A. Strumia and N. Tetradis, Phys. Rev. D 77 (2008) 025034.
b. A. Salvio, A. Strumia, N. Tetradis and A. Urbano, JHEP 1609 (2016) 054

Decay Rate: Higgs Potential



$$\lambda < 0$$

$$V(\phi) = \frac{\lambda}{4}\phi^4$$

$$\phi \to \frac{1}{\alpha}\phi \qquad \rho \to \alpha\rho$$

$$\Gamma \simeq \frac{V_U}{R^4} e^{-\frac{8\pi^2}{3|\lambda|}}$$

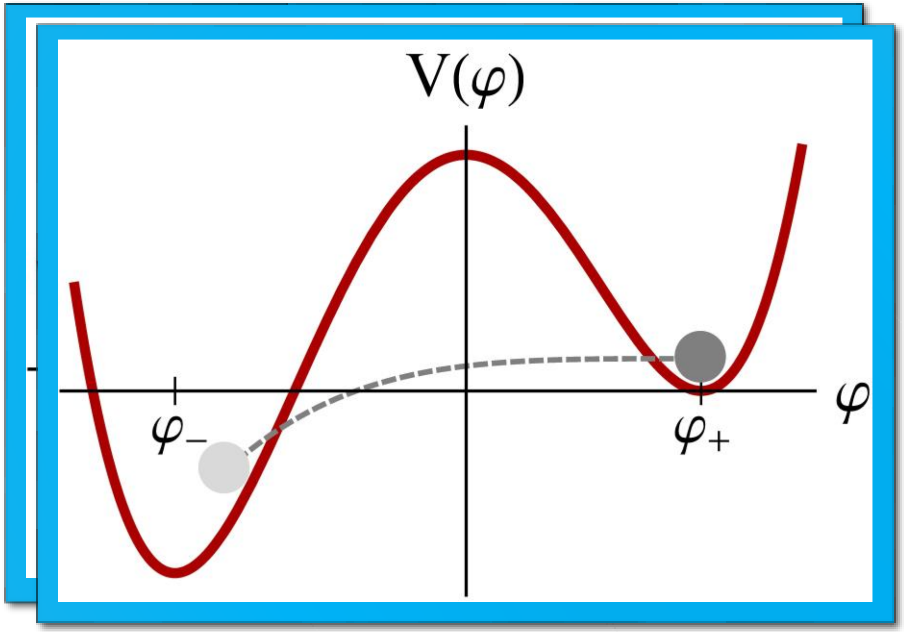
$$\Gamma \simeq \left(10^{25} eV\right)^4 \left(8.10^{101} eV^{-3}\right) e^{-\frac{8\pi^2}{3|-0.02|}} \simeq 10^{-370} eV$$

$$t \equiv \frac{1}{\Gamma} \simeq 10^{347} \text{ years}$$

a. K. M. Lee and E. J. Weinberg, Nucl. Phys. B 267, 181 (1986).

Single Field False Vacuum Decay

$$S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V \right]$$



$$\frac{\partial\varphi}{\partial\tau}(0,\vec{x}) = 0$$

$$\lim_{\tau\to\pm\infty} \varphi(\tau,\vec{x}) = \varphi_+$$

$$\lim_{\vec{x}\to\pm\infty} \varphi(\tau,\vec{x}) = \varphi_+$$

Solving Boundary Conditions

$$\sum_{\sigma=0}^{N-1} (a_{\sigma+1} - a_{\sigma}) R_{\sigma}^{D} = 0$$

$$S_D^{(\lambda)} = \lambda^{D-2}\mathcal{T} + \lambda^D \mathcal{V}$$

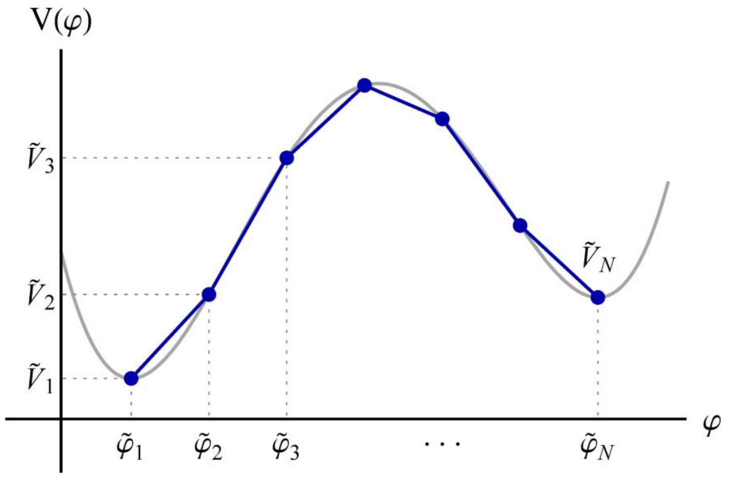
$$\left.\frac{dS_D^{(\lambda)}}{d\lambda}\right|_{\lambda=1} = 0 \qquad (D-2)\mathcal{T} + D\mathcal{V} = 0$$

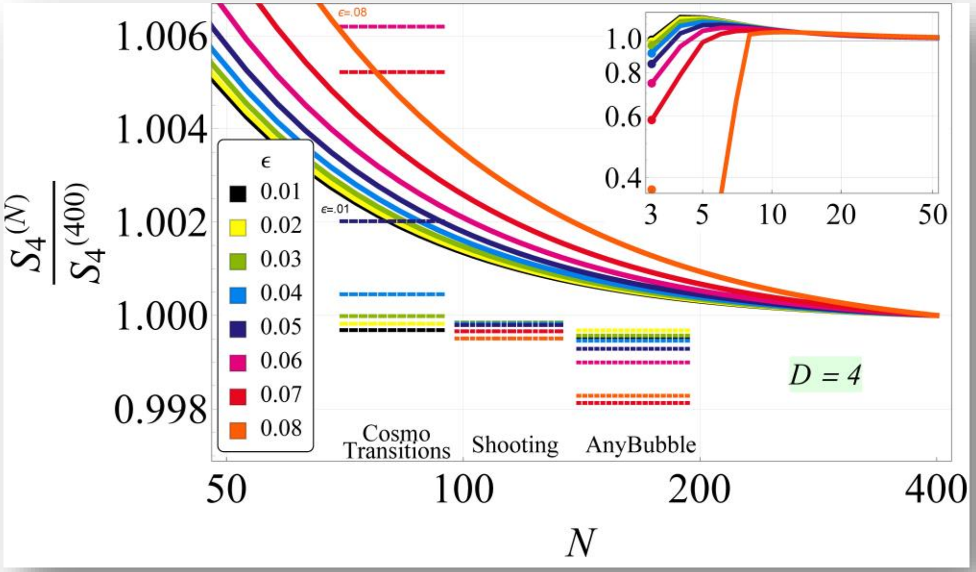
$$\lambda = \sqrt{\frac{(2-D)\mathcal{T}}{D\mathcal{V}}} = 1$$

a. G. H. Derrick, J. Math. Phys. 5 (1964) 1252.

Coleman Potential

$$V(\varphi) = \frac{\lambda}{8}\left(\varphi^2 - v^2\right)^2 + \varepsilon\left(\frac{\varphi - v}{2v}\right)$$





$$\mathcal{I}(\rho) = \int_{\rho_0}^{\rho} \mathrm{d}y\, y^{1-D} \int_{\rho_1}^{y} \mathrm{d}x\, x^{D-1} \delta dV(x)$$

$$\delta dV = dV(\varphi_{PB}(\rho)) - 8\,(a+\alpha)$$

Expansions

$$\tilde{V}_s - \tilde{V}_N + \partial\tilde{V}_s\,(\varphi_s - \tilde{\varphi}_s) + \frac{\partial^2 \tilde{V}_s}{2}\,(\varphi_s - \tilde{\varphi}_s)^2 + \dots$$

$$R_s \to R_s\,(1 + r_s) \qquad r_s \ll 1$$

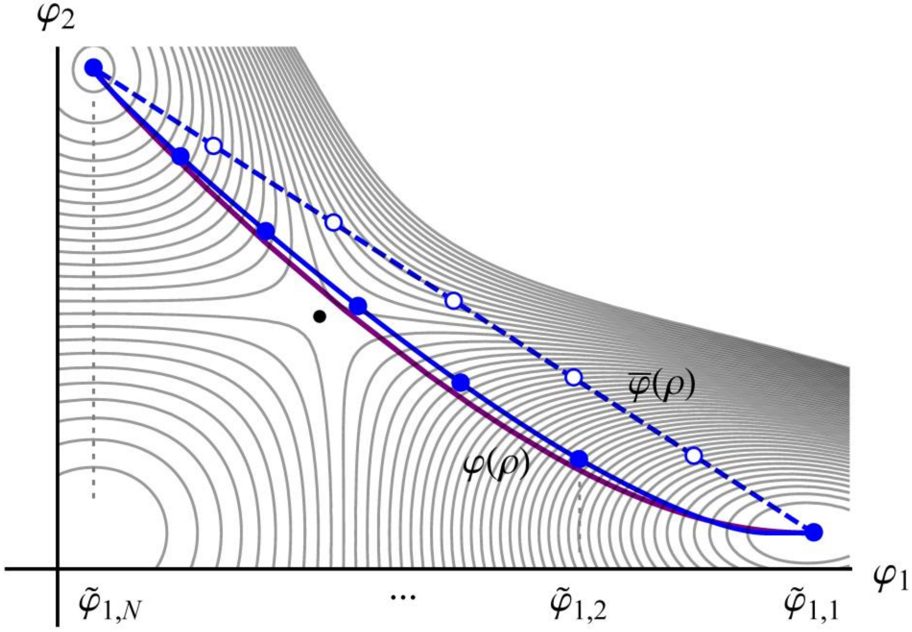
$$\dot{\varphi}_s(R_s) = \dot{\varphi}_{s+1}(R_s)$$

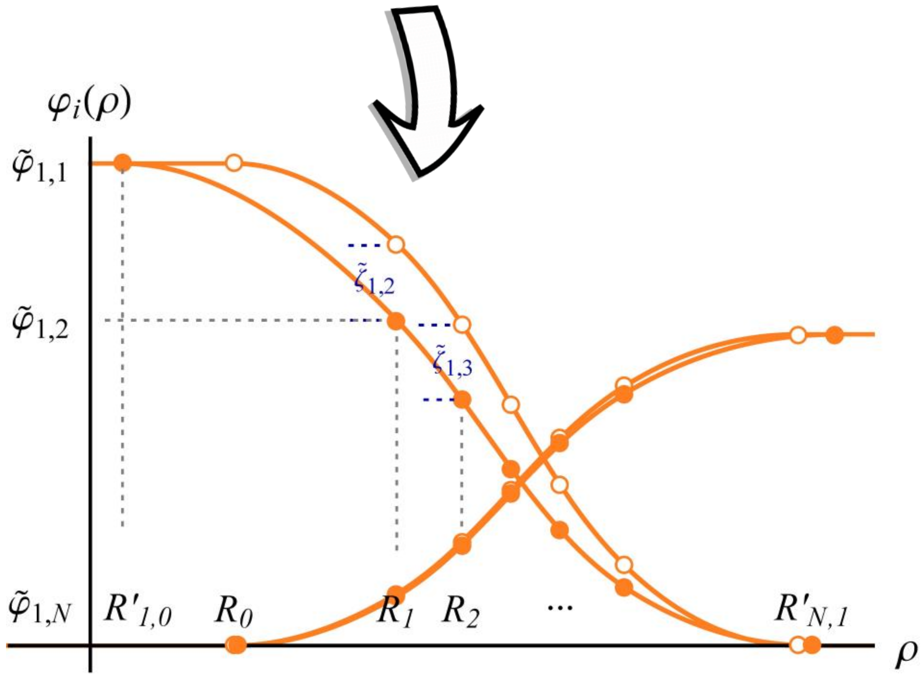
$$\varphi_s(R_s) = \tilde{\varphi}_{s+1} = \varphi_{s+1}(R_s)$$

Boundary Conditions

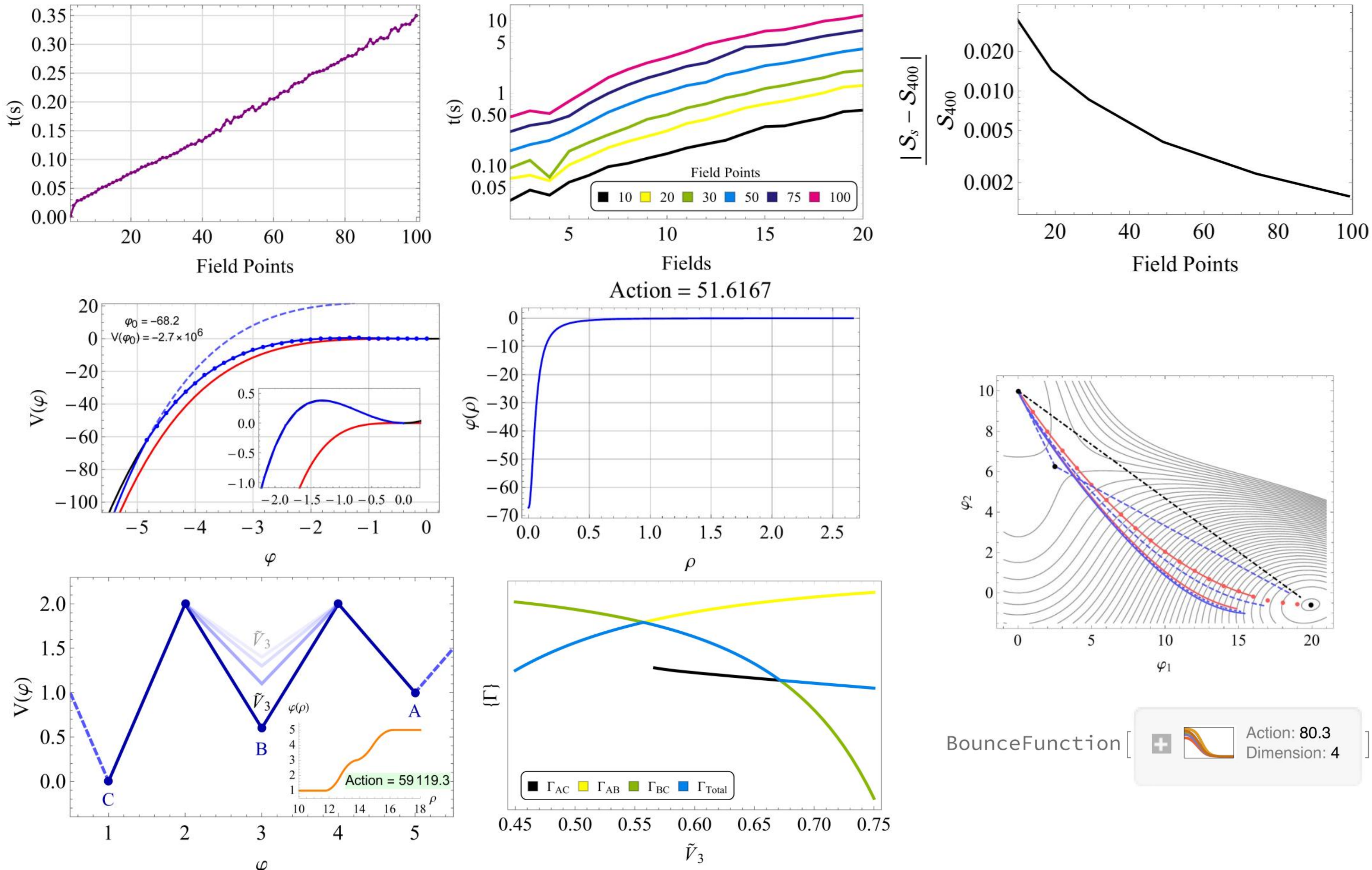
$$\zeta_{is}(R_s) = \tilde{\zeta}_{is+1}$$
$$R_{iN-1} = R_{N-1}\left(1 + r_{iN-1}\right)$$
$$R_{i0} = R_0\left(1 + r_{i0}\right)$$

$$\zeta_{is} = v_{is} + \frac{2}{D-2}\frac{b_{is}}{\rho^{D-2}} + \frac{4}{D}a_{is}\rho^2$$

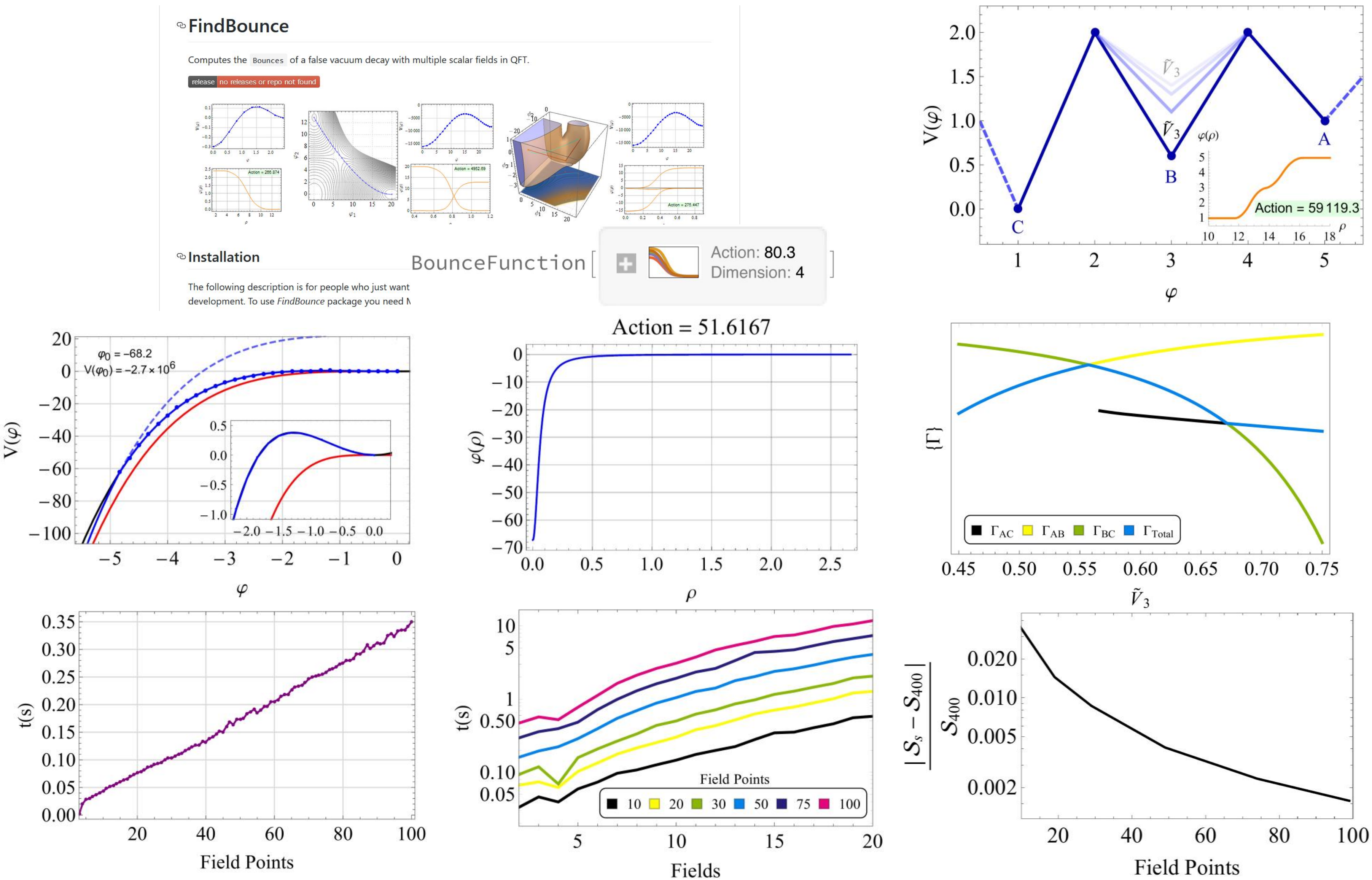




FindBounce: package



FindBounce



FindBounce
Computes the Bounces of a false vacuum decay with multiple scalar fields in QFT.
release no releases or repo not found
Installation
The following description is for people who just want
development. To use FindBounce package you need
BounceFunction[Action: 80.3 Dimension: 4]
V(φ)
φ
A
B
C
Ṽ3
φ(ρ)
ρ
Action = 59 119.3
φ0 = −68.2
V(φ0) = −2.7 × 10^6
Action = 51.6167
{Γ}
ΓAC ΓAB ΓBC ΓTotal
Ṽ3
t(s)
Field Points
Fields
Field Points
10 20 30 50 75 100
|Ss − S400| / S400