# Nonperturbative thermodynamics of multi-Higgs models

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Based on 1904.01329, 1903.11604

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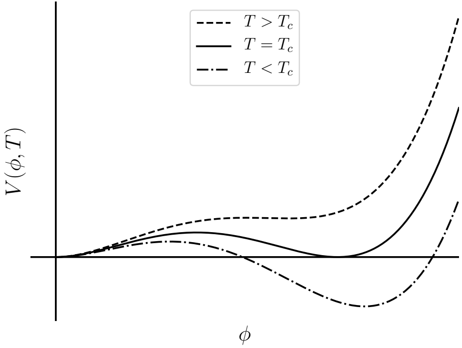
# First-order phase transitions in QFT

- $T = 0$ effective potential + thermal corrections:



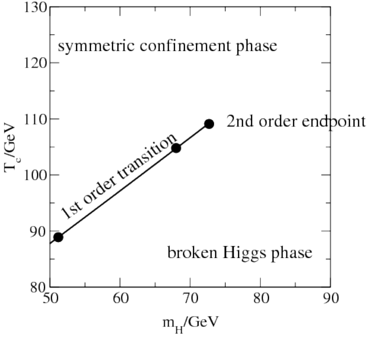

- Energy minima separated by a potential barrier $\rightarrow$ latent heat from tunneling

# First order electroweak phase transition?

NO phase transition in the SM (smooth crossover)!

Need significant modifications
to EW scale dynamics from BSM physics:

- Through radiative corrections
  to the Higgs (often $\mathcal{O}(1)$ couplings)?
- Multiple phase transitions
  before settling to the EW minimum?
- Non-renormalizable operators?




Source: hep-ph/0010275

# Why not perturbation theory?

- Infrared problem: light bosons are nonperturbative

$$\text{expansion parameter} \sim g^2 n_b(m) = \frac{g^2}{e^{m/T} - 1} \xrightarrow{m \ll T} \frac{g^2 T}{m}$$

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Examples:

- Perturbation theory underestimates transition strength by $\sim 50\%$ in MSSM    Laine et al. '12
- Perturbative effective potential always has a barrier

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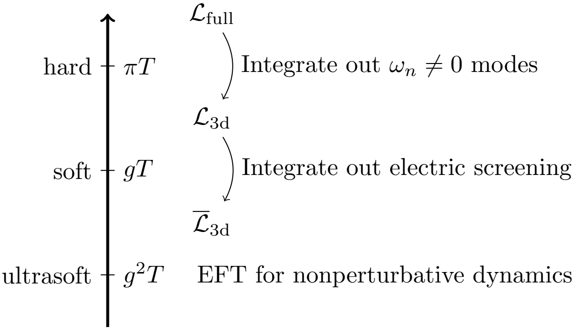
- What are the heavy modes?
  $\rightarrow$Thermal fluctuations of inverse length $\lesssim \pi T$:

$$\phi(\tau, \mathbf{x}) = T \sum_n \tilde{\phi}(\mathbf{p}) e^{i\omega_n \tau}, \ \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

Matsubara decomposition

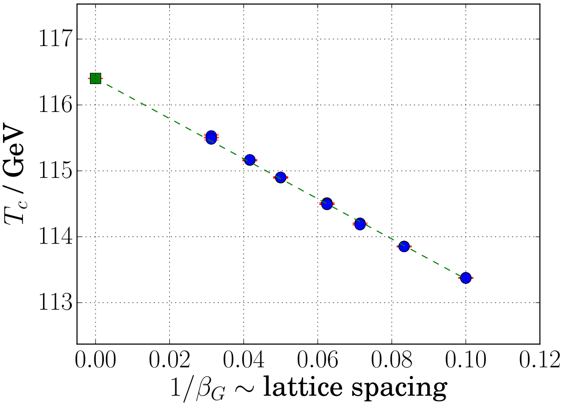
# High-$T$ effective theories

- Temperature decouples →IR theory is three dimensional and easy to simulate
- Easily applied to (perturbative) BSM theories



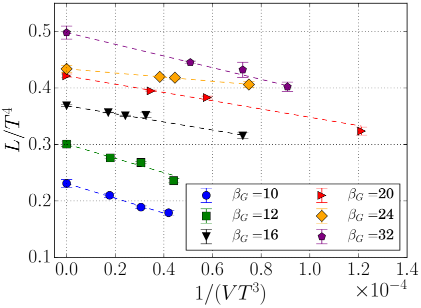
# Lattice simulations provide accurate results

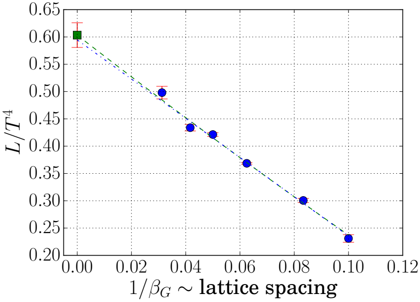
Example: phase transition with two Higgs doublets Kainulainen et al. 1904.01329



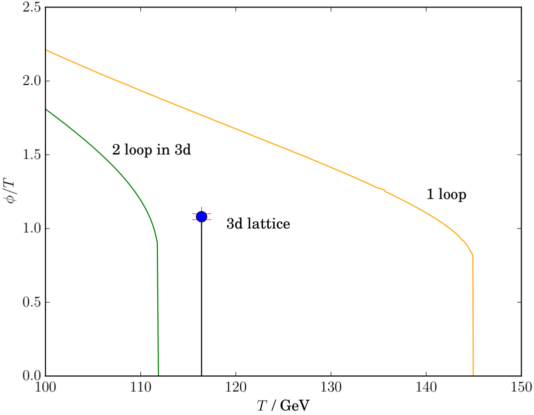
# Continuum extrapolation for latent heat

Measurable from discontinuity of condensates $\langle \phi^\dagger \phi \rangle$, $\langle (\phi^\dagger \phi)^2 \rangle$, ...



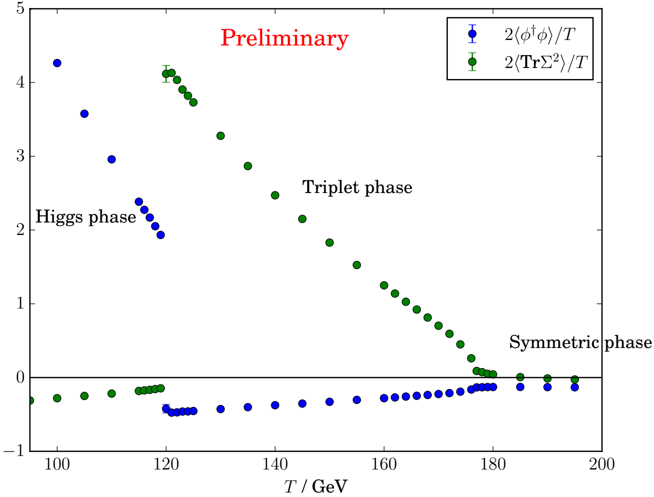



# Comparison with perturbation theory



# Multiple phase transitions?

SM + $\mathrm{SU}(2)$ triplet scalar (finite lattice):



# Model independent study?

EFT for SM + any sufficiently heavy scalar:

$$\mathcal{L}_{\text{3d}} = \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\phi)^\dagger(D_i\phi) + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2 + \text{neglected higher-order operators}$$

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$$+ \text{ neglected higher-order operators}$$

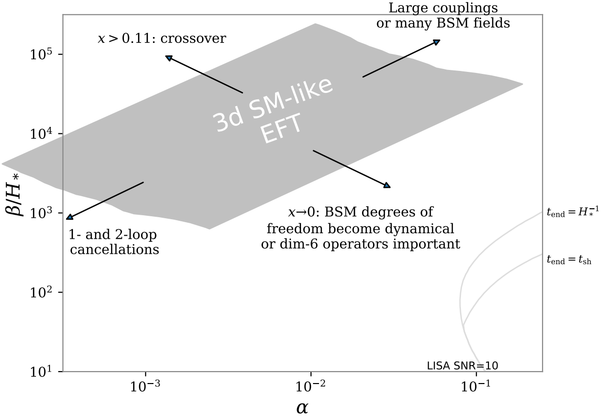
Nonperturbative phase diagram known Kajantie et al. ’96

$$x \equiv \frac{\lambda_3}{g_3^2}, \quad y \equiv \frac{m_3^2}{g_3^4} \approx 0 \text{ at } T_c$$

- First order transition for $0 < x < 0.11$
- Crossover for $x > 0.11$

# Gravitational waves from the SM-like EFT

Nonperturbative nucleation rate known    Moore & Rummukainen ’01




No observable gravitational wave signals from any theory that looks like SM in the infrared at high-$T$!    arXiv:1903.11604

# Conclusions

- New physics contributions can turn the EW phase transition into first order
- Calculations at high temperature can be simplified with effective theory methods
- Lattice methods can provide quantitative results, and can be used to estimate the performance of perturbation theory

# Backup: Properties of the effective theory

Example: Two Higgs doublet model.

$$V(\phi_1, \phi_2) = m_1^2 \phi_1^\dagger \phi_1 + m_2^2 \phi_2^\dagger \phi_2 + m_{12}^2 \phi_1^\dagger \phi_2 + \cdots + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \ldots$$

The resulting EFT:

$$\begin{aligned} \bar{\mathcal{L}}_{\text{3d}} =& \frac{1}{4}(F_{rs})^2 + (D_r \phi_1)^\dagger (D_r \phi_1) + (D_r \phi_2)^\dagger (D_r \phi_2) + \overline{m}_1^2(T) \phi_1^\dagger \phi_1 \\ &+ \overline{m}_2^2(T) \phi_2^\dagger \phi_2 + \overline{m}_{12}^2(T) \phi_1^\dagger \phi_2 + \cdots + \overline{\lambda}_3(T) (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \ldots \end{aligned}$$

- Super-renormalizable $\rightarrow$ **exact** relations to lattice parameters.
- Very accurate in the weak coupling regime. Error in the SM is $\sim 1\%$.

# Backup: Finding the transition point from histograms

- Find precise $T_c$ by reweighting the histograms to equal weight.