Scalar leptoquark in nucleon decays

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- electric charge not quantized
- many free parameters
- why 3 fermion famillies?
- neutrino masses?

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- one force
- one coupling constant

Motivation

the Standard Model

- electric charge not quantized
- many free parameters
- why 3 fermion famillies?
- neutrino masses?

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Grand Unified Theories

- one force
- one coupling constant

Grand Unified Theories

- quarks and leptons in the same multiplets
  $\Rightarrow$ electric charge quantized
  $\Rightarrow$ new interactions between quarks and leptons
  $\Rightarrow$ barion number violation
  $\Rightarrow$ proton decay ($\tau_p > 10^{30}$ y)
- $m_X \sim 10^{14-16}$ GeV

Leptoquarks

$(SU(3)_{\mathrm{C}}, SU(2)_{\mathrm{L}}, U(1)_{\mathrm{Y}})$	spin	symbol	coupling (quark and lepton)	coupling (pair of quarks)	F
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	LL	LL	-2
$(\mathbf{3}, \mathbf{2}, 7/6)$		R_2	RL, LR		0
$(\mathbf{3}, \mathbf{2}, 1/6)$		$\tilde{R}_2$	RL, $\overline{\mathrm{LR}}$		0
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$		$\tilde{S}_1$	RR	RR	-2
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$		S_1	LL, RR, $\overline{\mathrm{RR}}$	LL, RR	-2
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$		$\overline{S}_1$	$\overline{\mathrm{RR}}$	RR	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	LL		0
$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$		V_2	RL, LR	LR	-2
$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$		$\tilde{V}_2$	RL, $\overline{\mathrm{LR}}$	RL	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$		$\tilde{U}_1$	RR		0
$(\mathbf{3}, \mathbf{1}, 2/3)$		U_1	LL, RR, $\overline{\mathrm{RR}}$		0
$(\mathbf{3}, \mathbf{1}, -1/3)$		$\overline{U}_1$	$\overline{\mathrm{RR}}$		0

Leptoquarks

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$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	LL	LL	-2
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$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$		S_1	LL, RR, $\overline{\mathrm{RR}}$	LL, RR	-2
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$		$\bar{S}_1$	$\overline{\mathrm{RR}}$	RR	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	LL		0
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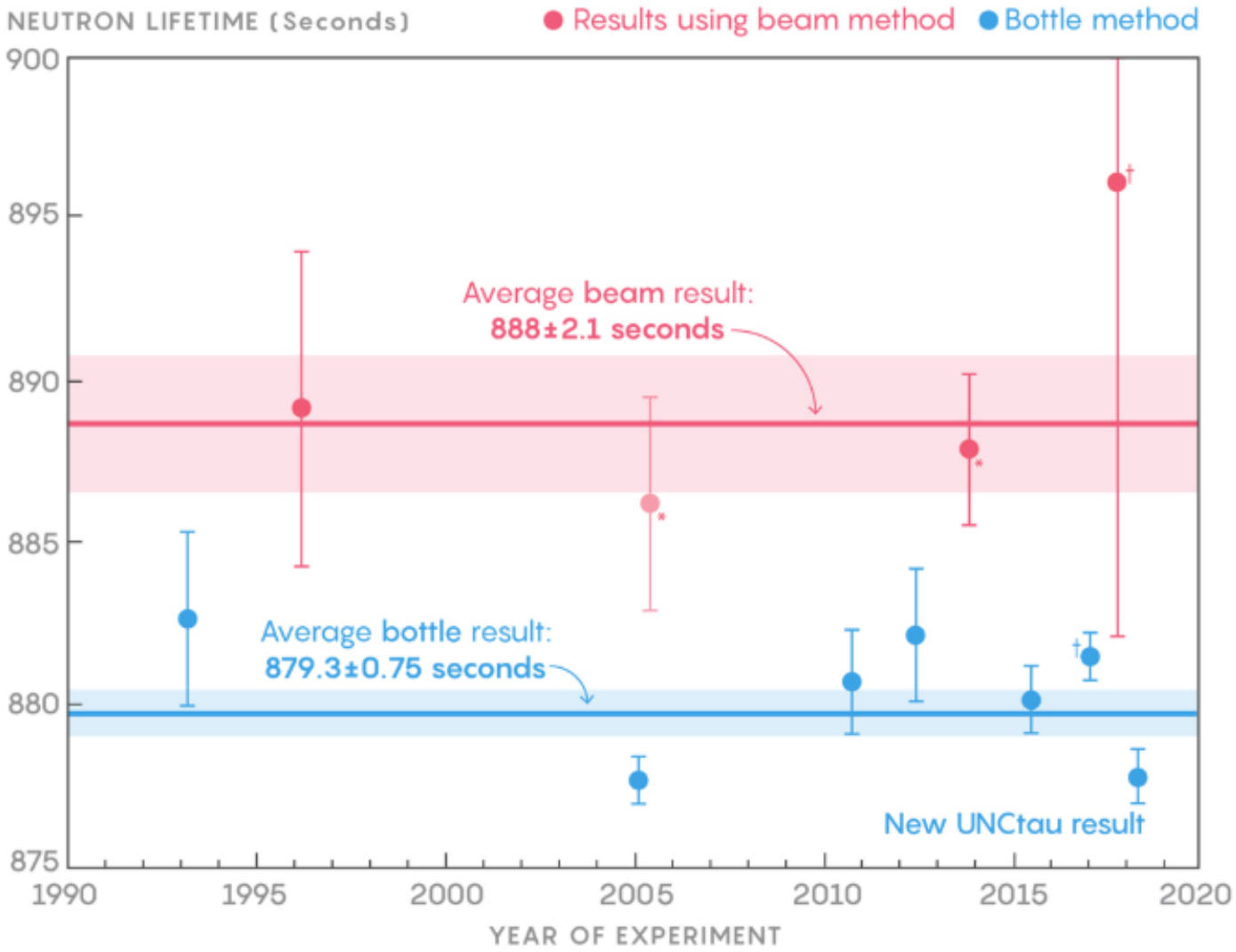
$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$		$\tilde{V}_2$	RL, $\overline{\mathrm{LR}}$	RL	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$		$\tilde{U}_1$	RR		0
$(\mathbf{3}, \mathbf{1}, 2/3)$		U_1	LL, RR, $\overline{\mathrm{RR}}$		0
$(\mathbf{3}, \mathbf{1}, -1/3)$		$\bar{U}_1$	$\overline{\mathrm{RR}}$		0

Discrepancy between the neutron lifetime measurements



$$\Delta\tau_n = 8.7 \pm 2.2\,\mathrm{s}$$

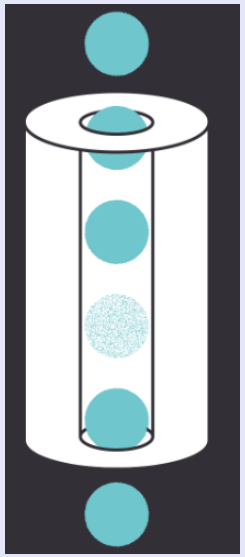
Two different methods

bottle method

$$\Gamma_n = \frac{\hbar}{\tau_n^{\text{bottle}}}$$

beam method

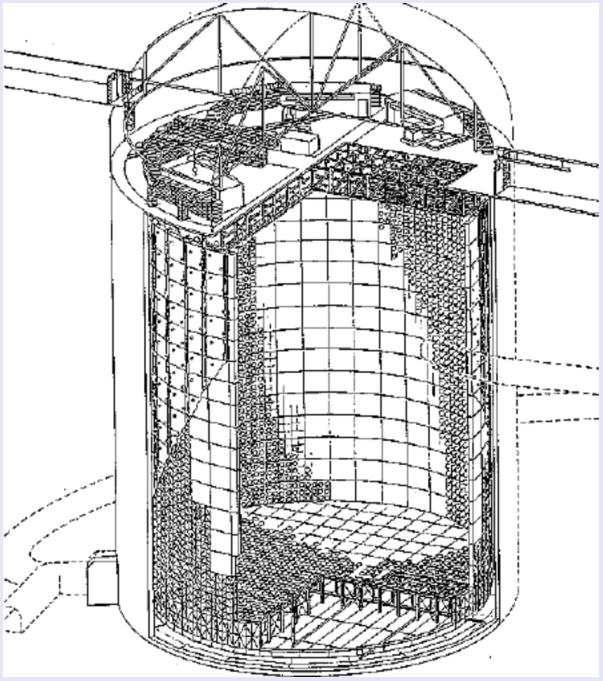
$$\Gamma_n^{\text{SM}} = \Gamma(n \to pe^-\overline{\nu}_e) = \frac{\hbar}{\tau_n^{\text{beam}}}$$



$$\Gamma_n^{\text{BSM}} = \Gamma(n \to \chi\gamma) = \Gamma_n - \Gamma_n^{\text{SM}} = \text{Br}(n \to \chi\gamma)\Gamma_n\,, \quad \text{Br}(n \to \chi\gamma) \approx 1\%$$

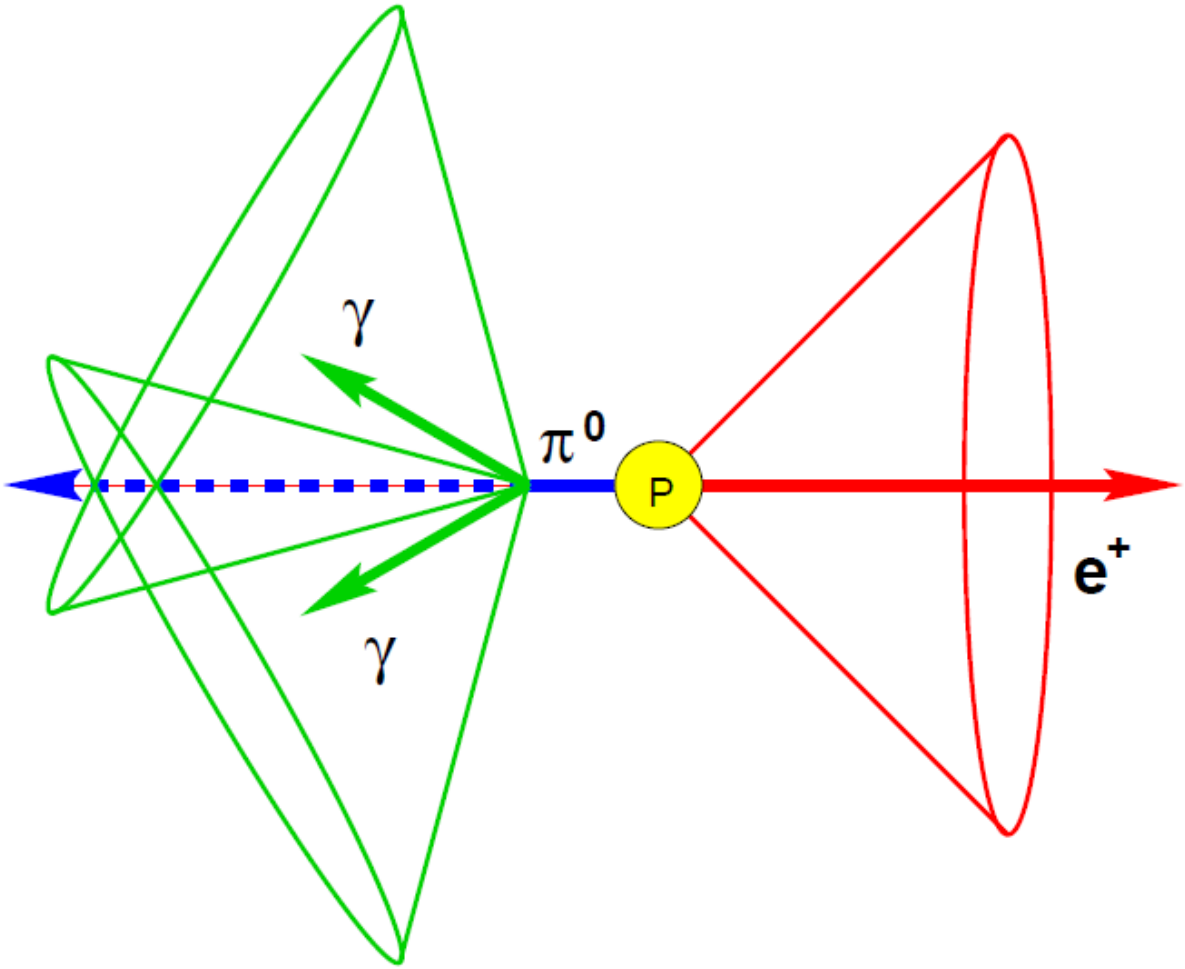
Proton lifetime measurements

Super-Kamiokande



Cherenkov radiation detection

$$p \rightarrow e^{+}\pi^{0}$$



Starting point: scalar leptoquark S_1

$$(SU(3)_{\mathrm{C}}, SU(2)_{\mathrm{L}}, U(1)_{\mathrm{Y}}): \qquad (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$$

$$\mathcal{L}_{S_1} = +y_{1\,ij}^{\mathrm{LL}} \overline{Q}_{\mathrm{L}}^{\mathrm{C}\,i,a} S_1 \varepsilon^{ab} L_{\mathrm{L}}^{j,b} + y_{1\,ij}^{\mathrm{RR}} \overline{u}_{\mathrm{R}}^{\mathrm{C}\,i} S_1 e_{\mathrm{R}}^{j} + y_{1\,ij}^{\overline{\mathrm{RR}}} \overline{d}_{\mathrm{R}}^{\mathrm{C}\,i} S_1 \chi^{j}$$
$$+ z_{1\,ij}^{\mathrm{LL}} \overline{Q}_{\mathrm{L}}^{\mathrm{C}\,i,a} S_1^{*} \varepsilon^{ab} Q_{\mathrm{L}}^{j,b} + z_{1\,ij}^{\mathrm{RR}} \overline{u}_{\mathrm{R}}^{\mathrm{C}\,i} S_1^{*} d_{\mathrm{R}}^{j} + \mathrm{h.\ c.}\,,$$

$$\Downarrow$$

$$\mathcal{L}_{S_1} = y_{1\,11}^{\mathrm{RR}} \overline{u}_{\mathrm{R}}^{\mathrm{C}} S_1 e_{\mathrm{R}} + y_{1\,11}^{\overline{\mathrm{RR}}} \overline{d}_{\mathrm{R}}^{\mathrm{C}} S_1 \chi + z_{1\,11}^{\mathrm{RR}} \overline{u}_{\mathrm{R}}^{\mathrm{C}} S_1^{*} d_{\mathrm{R}} + \mathrm{h.\ c.}$$

$$\uparrow$$

B. Fornal in B. Grinstein, *Dark Matter Interpretation of the Neutron Decay Anomaly*, Phys. Rev. Lett. **120**, 191801 (2018).

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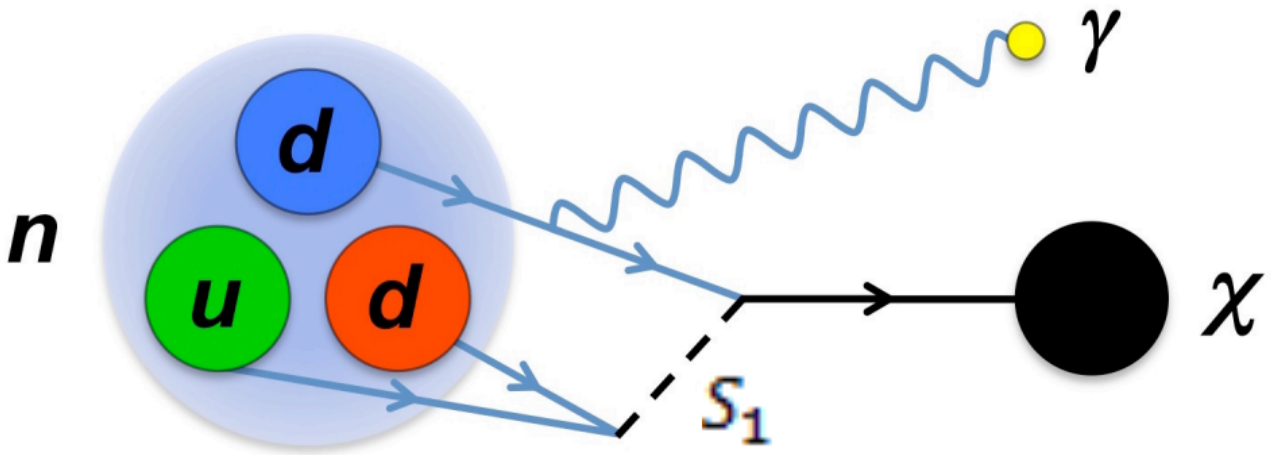
$n \to \chi\gamma$

$$\mathcal{L}_{S_1} = y_{111}^{\mathrm{RR}} \overline{u}_{\mathrm{R}}^{\mathrm{C}} S_1 e_{\mathrm{R}} + y_{111}^{\overline{\mathrm{RR}}} \overline{d}_{\mathrm{R}}^{\mathrm{C}} S_1 \chi + z_{111}^{\mathrm{RR}} \overline{u}_{\mathrm{R}}^{\mathrm{C}} S_1^* d_{\mathrm{R}} + \mathrm{h.\ c.}$$

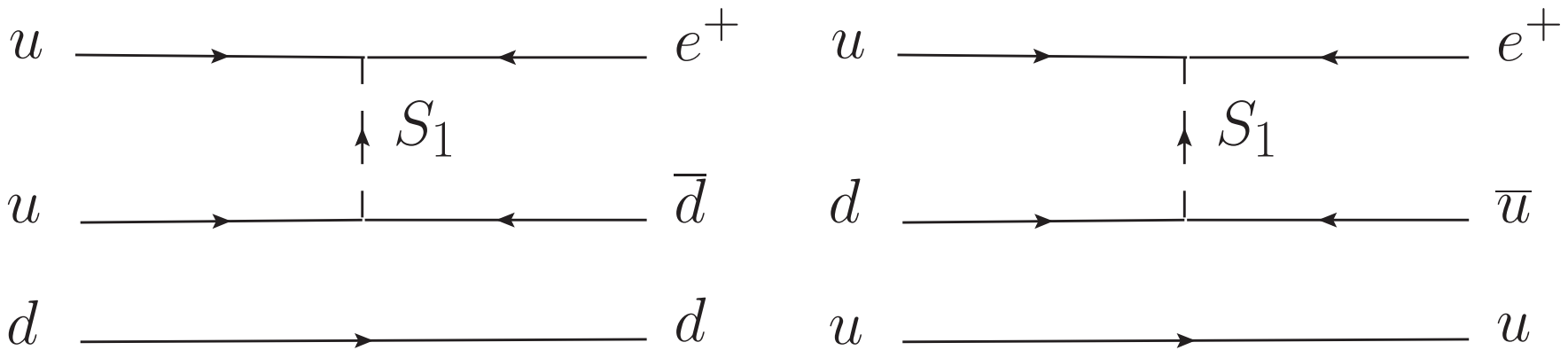
$$\Delta\tau_n = 8.6 \pm 2.1\,\mathrm{s}$$



$p \to e^+\pi^0$

$$\mathcal{L}_{S_1} = y_{111}^{\mathrm{RR}}\overline{u}_{\mathrm{R}}^{\mathrm{C}} S_1 e_{\mathrm{R}} + y_{111}^{\overline{\mathrm{RR}}}\overline{d}_{\mathrm{R}}^{\mathrm{C}} S_1 \chi + z_{111}^{\mathrm{RR}}\overline{u}_{\mathrm{R}}^{\mathrm{C}} S_1^* d_{\mathrm{R}} + \mathrm{h.\ c.}$$

$$\tau(p \to e^+\pi^0) > 1.6 \times 10^{34}\ \mathrm{y}$$



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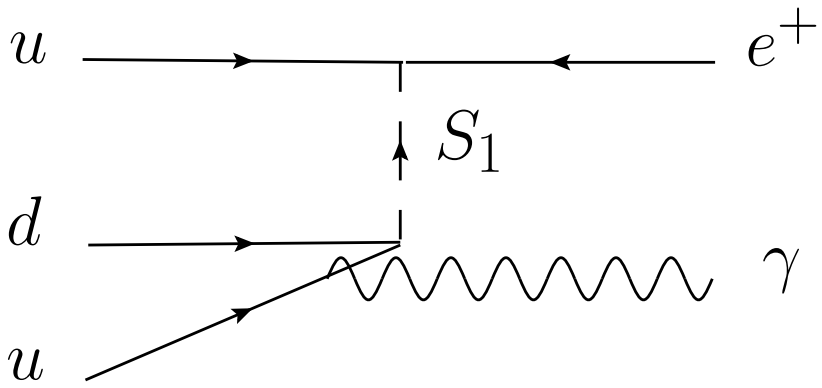
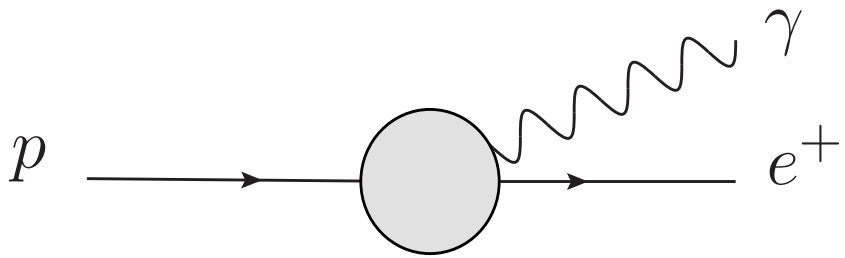
$$\tau(p \to e^+\gamma) > 6.7 \times 10^{32}\,\mathrm{y}$$





Atomic parity violation in cesium

weak charge: $Q_{\mathrm{W}}^{\mathrm{SM}}(Z,N) = -2(2Z+N)C_{1u} - 2(Z+2N)C_{1d}$

new physics contribution: $\delta C_{1q} = c_{qq;ee}^{\mathrm{LL}} - c_{qq;ee}^{\mathrm{LR}} + c_{qq;ee}^{\mathrm{RL}} - c_{qq;ee}^{\mathrm{RR}}$

$$\delta C_{1u} = -c_{11;11}^{\mathrm{RR}} = \frac{v^2}{4m_{S_1}^2}\left(y_{1\,11}^{\mathrm{RR}}\right)^2$$

weak charge measurement in ^{133}Cs differs from the SM:

$$\delta Q_{\mathrm{W}} = Q_{\mathrm{W}} - Q_{\mathrm{W}}^{\mathrm{SM}} = 0.65(43)$$

$$|\delta C_{1u}| = \left|\frac{\delta Q_{\mathrm{W}}}{376}\right| \sim 10^{-3}$$

$$\frac{\left|y_{1\,11}^{\mathrm{RR}}\right|}{m_{S_1}} \sim 2.6 \times 10^{-4}\,\mathrm{GeV}^{-1}$$

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$$\frac{\left|y_{1\,11}^{\mathrm{RR}}\right|}{m_{S_1}} \sim 2.6 \times 10^{-4}\ \mathrm{GeV}^{-1}$$

$\Gamma(p \to e^+\pi^0)$

$$\Gamma(p \to e^+\pi^0) = \frac{1}{8\pi}\overline{|\mathcal{M}|^2}\frac{|\boldsymbol{p}_{\mathrm{CM}}|}{m_p^2}$$

$$= \frac{1}{32\pi}\left(\frac{y_{1\,11}^{\mathrm{RR}} z_{1\,11}^{\mathrm{RR}}}{m_{S_1}^2}\right)^2 \left(W_0^{\mathrm{RR}}(0)\right)^2 \left(1 - \left(\frac{m_{\pi^0}}{m_p}\right)^2\right)^2 m_p$$

$\Gamma(n \to \chi\gamma)$

$$\mathcal{L}^{\text{eff}}_{N\gamma N} = e\overline{\psi}(x)\left[F_1(q^2)\gamma^\mu A_\mu(x) + \frac{1}{4m}F_2(q^2)\sigma^{\mu\nu}F_{\mu\nu}(x)\right]\psi(x)$$

$$\begin{aligned}\mathcal{L}^{\text{eff}}_1 &= \mathcal{L}^{\text{Dirac}}_{n'} + \mathcal{L}^{\text{Dirac}}_{\chi'} + \mathcal{L}^{\text{eff}}_{n'\gamma n'} + \mathcal{L}^{\text{eff}}_{n'\leftrightarrow\chi'} \\ &= \overline{n}'(i\partial\!\!\!/ - m_n)n' + \overline{\chi}'(i\partial\!\!\!/ - m_\chi)\chi' + \frac{ea_n}{4m_n}\overline{n}'\sigma^{\mu\nu}F_{\mu\nu}n' + \varepsilon(\overline{n}'\chi' + \overline{\chi}'n')\end{aligned}$$

mass matrix diagonalisation ($\varepsilon \ll m_n - m_\chi$) :

$$-m_n\overline{n}'n' - m_\chi\overline{\chi}'\chi' + \varepsilon(\overline{n}'\chi' + \overline{\chi}'n') = \begin{bmatrix}\overline{n}' & \overline{\chi}'\end{bmatrix}\begin{bmatrix}-m_n & \varepsilon \\ \varepsilon & -m_\chi\end{bmatrix}\begin{bmatrix}n' \\ \chi'\end{bmatrix}$$

$\Gamma(n \to \chi\gamma)$

mass eigenstates:

$$n = -n' + \frac{\varepsilon}{m_n - m_\chi}\chi' \,,$$
$$\chi = \frac{\varepsilon}{m_n - m_\chi} n' + \chi'$$

effective Lagrangian in the mass basis:

$$\mathcal{L}_1^{\text{eff}} = \mathcal{L}_n^{\text{Dirac}} + \mathcal{L}_\chi^{\text{Dirac}} + \mathcal{L}_{n\gamma n}^{\text{eff}} + \mathcal{L}_{n\gamma\chi}^{\text{eff}}$$
$$\mathcal{L}_{n\gamma\chi}^{\text{eff}} = -\frac{e a_n}{4 m_n} \frac{\varepsilon}{(m_n - m_\chi)} \overline{\chi} \sigma^{\mu\nu} F_{\mu\nu} n + \text{h. c.}$$

$\Gamma(n \to \chi\gamma)$

$$\Gamma(n \to \chi\gamma) = \frac{e^2 a_n^2}{32\pi} \frac{m_n \varepsilon^2}{(m_n - m_\chi)^2} \left(1 - \left(\frac{m_\chi}{m_n}\right)^2\right)^3$$

$\Gamma(n \to \chi\gamma)$

$$
\begin{aligned}
-i\mathcal{M} &= \langle\chi| \left(i z^{\mathrm{RR}}_{1\,11} \overline{u}^{\mathrm{C}}_{\mathrm{R}} d_{\mathrm{R}}\right) \frac{-i}{m^2_{S_1}} \left(i y^{\overline{\mathrm{RR}}}_{1\,11} \overline{d}^{\mathrm{C}}_{\mathrm{R}} \chi\right) |n\rangle \\
&= i \frac{y^{\overline{\mathrm{RR}}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}} P_{\mathrm{R}} v_\chi \langle 0| \left(\overline{u}^{\mathrm{C}} P_{\mathrm{R}} d\right) \overline{d}^{\mathrm{C}} P_{\mathrm{R}} |n\rangle \\
&= i \frac{y^{\overline{\mathrm{RR}}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}} \beta \left(\overline{v}_n P_{\mathrm{R}} v_\chi\right)
\end{aligned}
$$

$$
\begin{aligned}
-i\mathcal{M} &= \left\langle \chi \left| i\mathcal{L}^{\mathrm{eff}}_{n\to\chi} \right| n \right\rangle \\
&= \left\langle \chi \left| i\varepsilon \overline{\chi} n \right| n \right\rangle \\
&= i\varepsilon(\overline{u}_\chi P_{\mathrm{R}} u_n)
\end{aligned}
$$

mixing parameter: $\varepsilon = \dfrac{y^{\overline{\mathrm{RR}}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}} \beta\,, \qquad \beta = 0.0144(3)(21)\,\mathrm{GeV}^3$

$$
\Gamma(n \to \chi\gamma) = \frac{e^2 a_n^2}{32\pi} \left(\frac{y^{\overline{\mathrm{RR}}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}}\right)^2 \beta^2 \left(1 - \left(\frac{m_\chi}{m_n}\right)^2\right)^3 \frac{m_n}{(m_n - m_\chi)^2}
$$

$\Gamma(p \to e^+\gamma)$

interaction basis: $\mathcal{L}_2^{\text{eff}} = \mathcal{L}_{p'}^{\text{Dirac}} + \mathcal{L}_{e^{+\prime}}^{\text{Dirac}} + \mathcal{L}_{p'\gamma p'}^{\text{eff}} + \mathcal{L}_{e^{+\prime}\gamma e^{+\prime}}^{\text{eff}} + \mathcal{L}_{p' \leftrightarrow e^{+\prime}}^{\text{eff}}$

$$\mathcal{L}_{p'\gamma p'}^{\text{eff}} = e\overline{p}' \left[\gamma^\mu A_\mu + \frac{a_p}{4m_p} \sigma^{\mu\nu} F_{\mu\nu} \right] p' \,,$$

$$\mathcal{L}_{e^{+\prime}\gamma e^{+\prime}}^{\text{eff}} = e \left(\overline{e}^{+\prime} \gamma^\mu A_\mu e^{+\prime} \right)$$

mass basis: $\mathcal{L}_2^{\text{eff}} = \mathcal{L}_p^{\text{Dirac}} + \mathcal{L}_{e^+}^{\text{Dirac}} + \mathcal{L}_{p\gamma p}^{\text{eff}} + \mathcal{L}_{e^+\gamma e^+}^{\text{eff}} + \mathcal{L}_{p\gamma e^+}^{\text{eff}}$

$$\mathcal{L}_{p\gamma e^+}^{\text{eff}} = -\frac{e a_p}{4m_p} \frac{\varepsilon}{(m_p - m_e)} \overline{e}^+ \sigma^{\mu\nu} F_{\mu\nu} p + \text{h. c.}$$

$$\Gamma(p \to e^+\gamma) = \frac{e^2 a_p^2}{32\pi} \left(\frac{y_{1\,11}^{\text{RR}} z_{1\,11}^{\text{RR}}}{m_{S_1}^2} \right)^2 \beta^2 \left(1 - \left(\frac{m_e}{m_p} \right)^2 \right)^3 \frac{m_p}{(m_p - m_e)^2}$$

χ mass

Fornal and Grinstein exclude proton decay channel

$$p \to n^* e^+ \nu_e \to \chi e^+ \nu_e \,.$$

In this case m_χ is constrained:

$$m_n > m_\chi > m_p - m_e$$

Coupling constants

we use the previous constraint on m_χ, results from atomic parity violation,

$$\Delta\tau_n = 8.6 \pm 2.1\,\mathrm{s}$$

and

$$\tau(p \to e^+\pi^0) > 1.6 \times 10^{34}\,\mathrm{y}$$

$$\Gamma(p \to e^+\pi^0) \propto \left(\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}}\right)^2, \qquad \Gamma(n \to \chi\gamma) \propto \left(\frac{y^{\overline{\mathrm{RR}}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}}\right)^2$$

$$\Downarrow$$

$$\frac{\left|y^{\overline{\mathrm{RR}}}_{1\,11}\right|}{m_{S_1}} > 7.0 \times 10^{16}\,\mathrm{GeV}^{-1}$$

$\tau(p \to e^+\gamma)$

$$\Gamma(p \to e^+\pi^0) \propto \left(\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}}\right)^2 , \qquad \Gamma(p \to e^+\gamma) \propto \left(\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}}\right)^2$$

$$\tau(p \to e^+\pi^0) > 1.6 \times 10^{34}\,\mathrm{y} , \qquad \tau(p \to e^+\gamma) > 6.7 \times 10^{32}\,\mathrm{y}$$

$$\Downarrow$$

$$\tau_{\mathrm{new}}(p \to e^+\gamma) > 4.0 \times 10^{36}\,\mathrm{y}$$

Mass of the leptoquark S_1

$$\Gamma(p \to e^+\pi^0) \propto \left(\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}} \right)^2$$

$$\tau(p \to e^+\pi^0) > 1.6 \times 10^{34}\,\mathrm{y}$$

Taking both coupling constants of order one

$$\Downarrow$$

$$m_{S_1} \gtrsim 10^{16}\,\mathrm{GeV}\,.$$

This is GUT energy scale.

Conclusion

Also decays of very common hadrons like proton and neutron can put constraints on hypothetical particles.

Conclusion

Thank you for your attention.

Backup

Backup - the Standard Model

$$\varphi' = U_{G'}\varphi$$
$$A' = U_{G'} A U_{G'}^{-1}$$
$$U_{U(1)_\mathrm{Y}} = e^{i\alpha(x)}$$
$$U_{SU(2)_\mathrm{L}} = e^{i\theta^a(x)\frac{\tau^a}{2}}$$
$$U_{SU(3)_\mathrm{C}} = e^{i\xi^b(x)\frac{\lambda^b}{2}}$$

$$Q\langle 0|\phi|0\rangle = 0$$
$$\mathcal{L}_3 \supset \frac{m_h^2}{2}h^2$$
$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$
$$\begin{bmatrix} Z_\mu \\ A_\mu \end{bmatrix} = \begin{bmatrix} \cos\theta_\mathrm{W} & -\sin\theta_\mathrm{W} \\ \sin\theta_\mathrm{W} & \cos\theta_\mathrm{W} \end{bmatrix} \begin{bmatrix} W_\mu^3 \\ B_\mu \end{bmatrix}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_\mathrm{S} f^{abc} G_\mu^b G_\nu^c$$
$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\varepsilon^{abc} W_\mu^b W_\nu^c$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\cos\theta_\mathrm{W} = \frac{g}{\sqrt{g^2 + g'^2}}$$
$$\sin\theta_\mathrm{W} = \frac{g'}{\sqrt{g^2 + g'^2}}$$
$$\tan\theta_\mathrm{W} = \frac{g'}{g}$$

$$D_\mu = \partial_\mu + ig_\mathrm{S} G_\mu^a \frac{\lambda^a}{2} + igW_\mu^b \frac{\tau^b}{2} + ig' YB_\mu$$

Backup - SM Lagrangian before the symmetry breaking

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$$

gauge kinetic term:

$$\mathcal{L}_1 = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^b_{\mu\nu} W^{b\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

fermion term:

$$\mathcal{L}_2 = \overline{L}_{\mathrm{L}i} i \not{D} L_{\mathrm{L}i} + \overline{e}_{\mathrm{R}i} i \not{D} e_{\mathrm{R}i} + \overline{Q}_{\mathrm{L}i} i \not{D} Q_{\mathrm{L}i} + \overline{u}_{\mathrm{R}i} i \not{D} u_{\mathrm{R}i} + \overline{d}_{\mathrm{R}i} i \not{D} d_{\mathrm{R}i}$$

scalar term:

$$\mathcal{L}_3 = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi)$$

Yukawa coupling term:

$$\mathcal{L}_4 = y^{(e)}_{ij} \overline{L}_{\mathrm{L}i} \phi e_{\mathrm{R}j} + y^{(u)}_{ij} \overline{Q}_{\mathrm{L}i} \tilde{\phi} u_{\mathrm{R}j} + y^{(d)}_{ij} \overline{Q}_{\mathrm{L}i} \phi d_{\mathrm{R}j} + \mathrm{h.\ c.}$$

Backup - Higgs mechanism

$$\text{Higgs potential: } V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

$$|\phi|^2 = \frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\langle 0|\phi|0\rangle = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

$$\phi(x) = e^{i\frac{1}{v}\tau_b\xi_b} \begin{bmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{bmatrix}$$

Backup - the Standard Model

$$\mathcal{L}_3 \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu$$

$$m_W = \frac{gv}{2}, \qquad m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}, \qquad \frac{m_W^2}{m_Z^2} = \cos^2\theta_{\mathrm{W}}$$

$$\mathcal{L}_4 \supset \frac{v}{\sqrt{2}}\left(y_{ij}^{(e)}\overline{e}_{\mathrm{L}i}e_{\mathrm{R}j} + y_{ij}^{(u)}\overline{u}_{\mathrm{L}i}u_{\mathrm{R}j} + y_{ij}^{(d)}\overline{d}_{\mathrm{L}i}d_{\mathrm{R}j} + \mathrm{h.\ c.}\right)$$

$$M_{ij}^{(f)} = -\frac{v}{\sqrt{2}}y_{ij}^{(f)}$$

$$f'_{\mathrm{L}i} = S_{ij}f_{\mathrm{L}j}, \qquad f'_{\mathrm{R}i} = T_{ij}f_{\mathrm{R}j}$$

$$\overline{f}'_{\mathrm{L}i}M_{ij}^{(f)}f'_{\mathrm{R}j} = \left[\overline{f}'_{\mathrm{L}k}S_{ki}\right]\left[S_{il}^\dagger M_{lm}^{(f)}T_{mj}\right]\left[T_{jn}^\dagger f'_{\mathrm{R}n}\right] = \overline{f}_{\mathrm{L}i}\left(M_{\mathrm{d}}^{(f)}\right)_{ij}f_{\mathrm{R}j}$$

$$S_{ik}^\dagger M_{kl}^{(f)}T_{lj} = \left(M_{\mathrm{d}}^{(f)}\right)_{ij}, \qquad S_{ik}^\dagger y_{kl}^{(f)}T_{lj} = \left(y_{\mathrm{d}}^{(f)}\right)_{ij}$$

Backup - interactions

$$\mathcal{L}_2 \supset -eQ^{(e)}\overline{e}_i \not{A} e_i - eQ^{(u)}\overline{u}_i \not{A} u_i - eQ^{(d)}\overline{d}_i \not{A} d_i$$

$$e = g\sin\theta_{\mathrm{W}} = g'\cos\theta_{\mathrm{W}}$$

$$A_\mu \overline{f}'_{\mathrm{Li}}\gamma^\mu f'_{\mathrm{Li}} = A_\mu \overline{f}_{\mathrm{Lj}} S^\dagger_{ji}\gamma^\mu S_{ik} f_{\mathrm{L}k} = A_\mu \overline{f}_{\mathrm{L}j}\delta_{jk}\gamma^\mu f_{\mathrm{Lk}} = A_\mu \overline{f}_{\mathrm{L}i}\gamma^\mu f_{\mathrm{L}i}\ ,$$

$$\mathcal{L}_2 \supset -\frac{g'}{2\cos\theta_{\mathrm{W}}}\overline{f}_i\left(g_{\mathrm{V}}^{(f)}\not{Z} + g_{\mathrm{A}}^{(f)}\not{Z}\gamma_5\right) f_i\ ,$$

$$g_{\mathrm{V}}^{(f)} = T_3^{(f)} - 2Q^{(f)}\sin^2\theta_{\mathrm{W}}, \qquad g_{\mathrm{A}}^{(f)} = T_3^{(f)}$$

Backup - interactions

$$\mathcal{L}_2 \supset -\frac{g}{\sqrt{2}}\left(J^{\mathrm{CC}}_{q\mu}W^{+\mu} + J^{\mathrm{CC}}_{l\mu}W^{+\mu} + \mathrm{h.\ c.}\right)$$

$$J^{\mathrm{CC}}_{q\mu} = \overline{u}'_{\mathrm{L}i}\gamma_\mu d'_{\mathrm{L}i} = \overline{u}_{\mathrm{L}j}S^{(u)\dagger}_{ji}\gamma_\mu S^{(d)}_{ik}d_{\mathrm{L}k} = \overline{u}_{\mathrm{L}j}V_{jk}\gamma_\mu d_{\mathrm{L}k}$$

$$V = S^{(u)\dagger}S^{(d)}$$

$$u'_{\mathrm{L}i} = V^\dagger_{ij}u_{\mathrm{L}j} \qquad \text{in} \qquad d'_{\mathrm{L}i} = d_{\mathrm{L}i}$$

$$J^{\mathrm{CC}}_{l\mu} = \overline{\nu}'_{\mathrm{L}i}\gamma_\mu e'_{\mathrm{L}i} = \overline{\nu}_{\mathrm{L}j}S^{(\nu)\dagger}_{ji}\gamma_\mu S^{(e)}_{ik}e_{\mathrm{L}k} = \overline{\nu}_{\mathrm{L}j}\delta_{jk}\gamma_\mu e_{\mathrm{L}k} = \overline{\nu}_{\mathrm{L}i}\gamma_\mu e_{\mathrm{L}i}$$

$$S^{(\nu)\dagger}_{ik}M^{(\nu)}_{kl}T^{(\nu)}_{lj} = \left(M^{(\nu)}_{\mathrm{d}}\right)_{ij} = \mathrm{diag}(0,0,0)$$

$$J^{\mathrm{CC}\dagger}_{l\mu} = \overline{e}'_{\mathrm{L}i}\gamma_\mu \nu'_{\mathrm{L}i} = \overline{e}_{\mathrm{L}j}S^{(e)\dagger}_{ji}\gamma_\mu S^{(\nu)}_{ik}\nu_{\mathrm{L}k} = \overline{e}_{\mathrm{L}j}\gamma_\mu U_{jk}\nu_{\mathrm{L}k}$$

$$\nu'_{\mathrm{L}i} = U_{ij}\nu_{\mathrm{L}j} \qquad \text{in} \qquad e'_{\mathrm{L}i} = e_{\mathrm{L}i}$$

$$\nu'_{\mathrm{L}} = \begin{bmatrix}\nu_e & \nu_\mu & \nu_\tau\end{bmatrix}^T_{\mathrm{L}} = U\begin{bmatrix}\nu_1 & \nu_2 & \nu_3\end{bmatrix}^T_{\mathrm{L}}, \quad e'_{\mathrm{L}} = e_{\mathrm{L}} = \begin{bmatrix}e^- & \mu^- & \tau^-\end{bmatrix}^T_{\mathrm{L}}$$

$$\mathcal{L}_2 \supset -\frac{g_{\mathrm{s}}}{2}\overline{q}_i\lambda^a_{ij}\gamma_\mu G^\mu_a q_j$$

Backup - charge conjugation and chirality

$$\psi^{\mathrm{C}} = C\gamma^0\psi^* = i\gamma^2\psi^*$$

$$\psi_{\mathrm{L}} = P_{\mathrm{L}}\psi = \frac{(1-\gamma_5)}{2}\psi, \qquad \psi_{\mathrm{R}} = P_{\mathrm{R}}\psi = \frac{(1+\gamma_5)}{2}\psi$$

$$\overline{\psi}_{\mathrm{L,R}} \equiv (P_{\mathrm{L,R}}\psi)^\dagger\gamma^0 = \psi^\dagger P_{\mathrm{L,R}}\gamma^0 = \overline{\psi}P_{\mathrm{R,L}}$$

$$\psi^{\mathrm{C}}_{\mathrm{L,R}} \equiv (\psi_{\mathrm{L,R}})^{\mathrm{C}} = (\psi^{\mathrm{C}})_{\mathrm{R,L}}$$

$$\overline{\psi}^{\mathrm{C}}_{\mathrm{L,R}} \equiv \overline{\psi^{\mathrm{C}}_{\mathrm{L,R}}} = \overline{(\psi_{\mathrm{L,R}})^{\mathrm{C}}} = \overline{(\psi^{\mathrm{C}})_{\mathrm{R,L}}} = \overline{\psi^{\mathrm{C}}}P_{\mathrm{L,R}}$$

Backup - Lagrangian of the rare meson decays

$$
\begin{aligned}
\mathcal{L}_{\overline{q}^i q^j \overline{\ell}\ell'} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}} \Big[& c^{\mathrm{LL}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{L}} \gamma^\mu q^j_{\mathrm{L}}\right) \left(\overline{\ell}_{\mathrm{L}} \gamma_\mu \ell'_{\mathrm{L}}\right) + c^{\mathrm{RR}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{R}} \gamma^\mu q^j_{\mathrm{R}}\right) \left(\overline{\ell}_{\mathrm{R}} \gamma_\mu \ell'_{\mathrm{R}}\right) \\
& + c^{\mathrm{LR}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{L}} \gamma^\mu q^j_{\mathrm{L}}\right) \left(\overline{\ell}_{\mathrm{R}} \gamma_\mu \ell'_{\mathrm{R}}\right) + c^{\mathrm{RL}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{R}} \gamma^\mu q^j_{\mathrm{R}}\right) \left(\overline{\ell}_{\mathrm{L}} \gamma_\mu \ell'_{\mathrm{L}}\right) \\
& + g^{\mathrm{RR}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{R}} q^j_{\mathrm{L}}\right) \left(\overline{\ell}_{\mathrm{R}} \ell'_{\mathrm{L}}\right) + h^{\mathrm{RR}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{R}} \sigma^{\mu\nu} q^j_{\mathrm{L}}\right) \left(\overline{\ell}_{\mathrm{R}} \sigma_{\mu\nu} \ell'_{\mathrm{L}}\right) \\
& + g^{\mathrm{LL}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{L}} q^j_{\mathrm{R}}\right) \left(\overline{\ell}_{\mathrm{L}} \ell'_{\mathrm{R}}\right) + h^{\mathrm{LL}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{L}} \sigma^{\mu\nu} q^j_{\mathrm{R}}\right) \left(\overline{\ell}_{\mathrm{L}} \sigma_{\mu\nu} \ell'_{\mathrm{R}}\right) \\
& + g^{\mathrm{LR}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{L}} q^j_{\mathrm{R}}\right) \left(\overline{\ell}_{\mathrm{R}} \ell'_{\mathrm{L}}\right) + g^{\mathrm{RL}}_{ij;\ell\ell'} \left(\overline{q}^i_{\mathrm{R}} q^j_{\mathrm{L}}\right) \left(\overline{\ell}_{\mathrm{L}} \ell'_{\mathrm{R}}\right) \Big] + \mathrm{h.\ c.}
\end{aligned}
$$

Backup - Wilson coefficients for the Lagrangian of the rare meson decays

$$c^{\mathrm{LL}}_{ij;\ell\ell'} = -\frac{v^2}{4m^2_{S_1}} \left(V^T y^{\mathrm{LL}}_1\right)_{j\ell'} \left(V^T y^{\mathrm{LL}}_1\right)^*_{i\ell}$$

$$c^{\mathrm{RR}}_{ij;\ell\ell'} = -\frac{v^2}{4m^2_{S_1}} y^{\mathrm{RR}}_{1\,j\ell'} y^{\mathrm{RR}*}_{1\,i\ell} \,,$$

$$g^{\mathrm{LL}}_{ij;\ell\ell'} = -4h^{\mathrm{LL}}_{ij;\ell\ell'} = \frac{v^2}{4m^2_{S_1}} y^{\mathrm{RR}}_{1\,j\ell'} \left(V^T y^{\mathrm{LL}}_1\right)^*_{i\ell}$$

$$g^{\mathrm{RR}}_{ij;\ell\ell'} = -4h^{\mathrm{RR}}_{ij;\ell\ell'} = \frac{v^2}{4m^2_{S_1}} \left(V^T y^{\mathrm{LL}}_1\right)_{j\ell'} y^{\mathrm{RR}*}_{1\,i\ell}$$

Backup - Atomic parity violation in cesium

$$\mathcal{L}_{\overline{q}^i q^j \overline{\ell}\ell'} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}}\left[c^{\mathrm{RR}}_{ij;\ell\ell'}\left(\overline{q}^i_{\mathrm{R}}\gamma^\mu q^j_{\mathrm{R}}\right)\left(\overline{\ell}_{\mathrm{R}}\gamma_\mu \ell'_{\mathrm{R}}\right)\ldots\right] + \mathrm{h.\ c.}$$

$$c^{\mathrm{RR}}_{ij;\ell\ell'} = -\frac{v^2}{4m^2_{S_1}} y^{\mathrm{RR}}_{1\,j\ell'} y^{\mathrm{RR}*}_{1\,i\ell} \quad \rightarrow \quad c^{\mathrm{RR}}_{11;11} = -\frac{v^2}{4m^2_{S_1}}\left(y^{\mathrm{RR}}_{1\,11}\right)^2 \quad (ue^- \rightarrow ue^-)$$

Interaction violating parity:

$$\mathcal{L}^{\mathrm{SM}}_{\mathrm{PV}} = \frac{G_{\mathrm{F}}}{\sqrt{2}}\sum_{q=u,d}\left[C_{1q}\left(\overline{e}\gamma^\mu\gamma^5 e\right)\left(\overline{q}\gamma_\mu q\right) + C_{2q}\left(\overline{e}\gamma^\mu e\right)\left(\overline{q}\gamma_\mu\gamma^5 q\right)\right]$$

Backup - Atomic parity violation in cesium

$$\mathcal{L}_{\overline{q}^i q^j \overline{\ell}\ell'} = -\frac{4G_F}{\sqrt{2}}\left[c^{RR}_{ij;\ell\ell'}\left(\overline{q}^i_R\gamma^\mu q^j_R\right)\left(\overline{\ell}_R\gamma_\mu \ell'_R\right)\ldots\right] + \text{h. c.}$$

$$c^{RR}_{ij;\ell\ell'} = -\frac{v^2}{4m^2_{S_1}} y^{RR}_{1\,j\ell'} y^{RR*}_{1\,i\ell} \quad \rightarrow \quad c^{RR}_{11;11} = -\frac{v^2}{4m^2_{S_1}}\left(y^{RR}_{1\,11}\right)^2 \quad (ue^- \rightarrow ue^-)$$

Interaction violating parity:

$$\mathcal{L}^{SM}_{PV} = \frac{G_F}{\sqrt{2}}\sum_{q=u,d}\left[C_{1q}\left(\overline{e}\gamma^\mu\gamma^5 e\right)\left(\overline{q}\gamma_\mu q\right) + C_{2q}\left(\overline{e}\gamma^\mu e\right)\left(\overline{q}\gamma_\mu\gamma^5 q\right)\right]$$

Backup - form factor W_0, β and the decay width differential

$$\left\langle P(k') \left| \mathcal{O}^{\Gamma\Gamma'}(q) \right| N(k,s) \right\rangle = \left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_N} W_1^{\Gamma\Gamma'}(q^2) \right] P_{\Gamma'} u_N(k,s)$$

$$\mathcal{O}^{\Gamma\Gamma'} = \left(\overline{q}^{\mathrm{C}} P_\Gamma q\right) P_{\Gamma'} q$$

$$\left\langle 0 \left| \left(\overline{u}^{\mathrm{C}} P_{\mathrm{R}} d\right) P_{\mathrm{R}} d \right| n \right\rangle = \beta P_{\mathrm{R}} u_n$$

$$\mathrm{d}\Gamma = \frac{1}{32\pi^2} \overline{|\mathcal{M}|^2} \frac{|\boldsymbol{p}_{\mathrm{CM}}|}{m^2} \mathrm{d}\Omega$$

$$\Gamma = \frac{1}{8\pi} \overline{|\mathcal{M}|^2} \frac{|\boldsymbol{p}_{\mathrm{CM}}|}{m^2}$$

Backup - completeness relation and traces

$$\{\gamma^5, \gamma^\mu\} = 0, \qquad (\gamma^5)^2 = \mathbb{I}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{I}$$

$$\sum_s u_a(p,s)\overline{u}_b(p,s) = \left(\not{p} + m\mathbb{I}\right)_{ab}$$

$$\sum_s v_a(p,s)\overline{v}_b(p,s) = \left(\not{p} - m\mathbb{I}\right)_{ab}$$

$$\sum_r \epsilon^*_\mu(q,r)\epsilon_\nu(q,r) = -g_{\mu\nu}$$

$$\mathrm{Tr}\left[\text{odd number } \gamma^\mu\right] = 0$$

$$\mathrm{Tr}\left[\gamma^5\right] = 0$$

$$\mathrm{Tr}\left[\gamma^5 \cdot \text{odd number } \gamma^\mu\right] = 0$$

$$\mathrm{Tr}\left[\gamma^5\gamma^\mu\gamma^\nu\right] = 0$$

$$\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\right] = 4g^{\mu\nu}$$

Backup - $\Gamma(p \to e^+\pi^0)$

$$
\begin{aligned}
-i\mathcal{M} &= \left\langle e^+(k_2)\pi^0(k_3) \middle| \left(iz^{\mathrm{RR}}_{1\,11}\overline{u}^{\mathrm{C}}_{\mathrm{R}} d_{\mathrm{R}}\right) \frac{i}{q'^2 - m^2_{S_1}} \left(iy^{\mathrm{RR}}_{1\,11}\overline{u}^{\mathrm{C}}_{\mathrm{R}} e_{\mathrm{R}}\right) \middle| p(k_1) \right\rangle \\
&= i\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}} P_{\mathrm{R}} v_{e^+}(k_2, s_2) \left\langle \pi^0(k_3) \middle| \left(\overline{u}^{\mathrm{C}} P_{\mathrm{R}} d\right) \overline{u}^{\mathrm{C}} P_{\mathrm{R}} \middle| p(k_1) \right\rangle \\
&= i\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}} W_0^{\mathrm{RR}}(k_2^2) \left(\overline{v}_p(k_1, s_1) P_{\mathrm{R}} v_{e^+}(k_2, s_2)\right)
\end{aligned}
$$

$$
\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{1}{2}\sum_{s_1, s_2} |\mathcal{M}|^2 \\
&= \left(\frac{y^{\mathrm{RR}}_{1\,11} z^{\mathrm{RR}}_{1\,11}}{m^2_{S_1}}\right)^2 \left(W_0^{\mathrm{RR}}(0)\right)^2 (k_2 \cdot k_1)
\end{aligned}
$$

Backup - magnetic moment and $\Gamma(n \to \chi\gamma)$

$$\text{proton:} \quad F_1^p(0) + F_2^p(0) = \mu_p = 1 + a_p = 1 + 1.793 = 2.793$$
$$\text{neutron:} \quad F_1^n(0) + F_2^n(0) = \mu_n = 0 + a_n = 0 - 1.913 = -1.913$$

$$-i\mathcal{M} = \frac{ea_n}{2m_n} \frac{\varepsilon}{(m_n - m_\chi)} q_\mu \epsilon_\nu^*(q,r) \overline{u}_\chi(k_2, s_2) \sigma^{\mu\nu} u_n(k_1, s_1)$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{2} \sum_{s_1, s_2, r} |\mathcal{M}|^2 = 2 \left(\frac{ea_n \varepsilon}{m_n(m_n - m_\chi)} \right)^2 (k_1 \cdot q)(k_2 \cdot q)$$

Backup - lower bounds on the partial proton lifetimes

proton decay mode	partial lifetime $[10^{30}\,\mathrm{y}]$
$p \to e^+\pi^0$	16000
$p \to \mu^+\pi^0$	7700
$p \to \nu\pi^+$	390
$p \to e^+\eta$	10000
$p \to \mu^+\eta$	4700
$p \to e^+\rho^0$	720
$p \to \mu^+\rho^0$	570
$p \to \nu\rho^+$	162
$p \to e^+\omega$	1600
$p \to \mu^+\omega$	2800
$p \to e^+K^0$	1000
$p \to \mu^+K^0$	1600
$p \to \nu K^+$	5900
$p \to e^+\gamma$	670
$p \to \mu^+\gamma$	478